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GRAVITOELECTROMAGNETISM

EXPLAINED BY

THE THEORY OF INFORMATONS

Edition 2021

About this book♣

This book focusses on the explanation of the gravitational interactions and phenomena as they are described and understood in the framework of *gravitoelectromagnetism* (*GEM*). GEM is a classical field theory, that is starting from the idea that the gravitational field must be isomorphic with the electromagnetic field in a vacuum. It is an extension of Newtonian gravity because it takes into account, in addition to their position, the kinematics of the gravitating objects. In this book it is shown that GEM perfectly can be explained by the "theory of informatons".
The theory of informatons develops the idea that any material object manifests itself in space by the emission - at a rate proportional to its rest mass - of *informatons*: mass and energy less granular entities rushing away with the speed of light and carrying information regarding the position and the velocity of their emitter. This implies that any material object is at the center of an expanding cloud of informatons that can be identified as the gravitational field linked to that object.
It is shown that the gravitational field is a dual entity always having a field- and an induction- component simultaneously created by their common sources: time-variable masses and mass flows, that the Maxwell-Heaviside equations are the expressions at the macroscopic level of the kinematics of the informatons, that the gravitational interaction is the effect of the fact that an object in a gravitational field tends to become "blind" for that field by accelerating according to a Lorentz-like law, and that an accelerated object is the source of gravitational radiation.

About the author

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♣ Slightly modified version of the publication in HADRONIC JOURNAL – volume 43 – number 2 – September 2020

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CHAPTER 1

INTRODUCTION

Daily contact with the things on hand confronts us with their *substantiality*. An object is not just form, it is also matter. It takes space, it eliminates emptiness. The amount of matter within the contours of a physical body is called its *mass*. The mass of an object manifests itself when it interacts with other objects. A fundamental form of interaction is "*gravitation*". Material objects ("*masses*") attract each other and, if they are free, they move to each other.

In the framework of *the classical theory of fields* ("*Newtonian gravity*"), the gravitational interactions are described by introducing the field concept. Each material object manifests its substantiality by creating and maintaining a vector field, characterized by the vectoral quantity $\vec{E}_{\boldsymbol{g}}$ that has a value at every point of space and time and is thus - relative to an inertial reference frame $\boldsymbol{O}$ - regarded as a function of space and time coordinates. And each object in that field experiences a tendency to accelerate. The field theory considers the gravitational field as the entity that *mediates* in the gravitational interactions.

Newtonian gravity is further developed and extended by Oliver Heaviside[1] and Oleg Jefimenko[2]. Their work results in the *theory of gravitoelectromagnetism* (*GEM*). In GEM the description of the gravitational field is starting from the idea that it must be isomorphic with the electromagnetic one. This implies that the gravitational field must be characterized by two vectoral quantities $\vec{E}_{\boldsymbol{g}}$ - the *gravitational field* or the *g-field* - and $\vec{B}_{\boldsymbol{g}}$ – the *gravitational induction* or the *g-induction* - that are analogue to respectively the electric field $\vec{E}$ and the magnetic induction $\vec{B}$. The gravitational induction $\vec{B}_{\boldsymbol{g}}$ is representative for the kinematics of the gravitating objects, a phenomenon that was not taken into account in Newtonian gravity. The starting point of GEM also implies that the relations between $\vec{E}_{\boldsymbol{g}}$ and $\vec{B}_{\boldsymbol{g}}$ (the GEM equations or the *Maxwell-Heaviside equations*) must be analogue to Maxwell's laws. Neither these equations nor their solutions indicate an existence of causal links between $\vec{E}_{\boldsymbol{g}}$ and $\vec{B}_{\boldsymbol{g}}$. Therefore, in the framework of GEM it must be concluded that a gravitational field is a dual entity always having a "field-" and an "induction-" component simultaneously created by their common sources: time-variable masses and mass flows.

Although GEM describes the gravitational phenomena in a correct and coherent manner, it doesn't create clarity about the physical nature of gravity: the gravitational field is considered as a purely mathematical construction. In what follows we develop the idea that, if masses can influence each other "at a distance", they must in one way or another exchange data. We assume that each mass emits information relative to its magnitude and its position, and that it is able to "interpret" the information emitted by its neighbours. In this way we propose a physical foundation of GEM by introducing *information* as the substance of a gravitational field[3],[4],[5],[6].

We start from the idea that a material object manifests itself in space by the emission - at a rate proportional to its rest mass - of mass and energy less granular entities that, relative to an inertial reference frame, are rushing away with the speed of light and are carrying information regarding the position ("*g-information*") and regarding the velocity ("*β-information*") of their emitter. Because they transport nothing than information, we call these entities "*informatons*". The gravitational field of a material object will then be understood as an expanding cloud of informatons, that forms an indivisible whole with that object.

In the *postulate of the emission of informatons*, we define an informaton by its attributes and determine the rules that govern the emission by a point mass that is anchored in an inertial reference frame ***O***.

The first consequence of that postulate is that a point mass at rest in ***O*** - and by extension any material object at rest - is the source of an expanding cloud of informatons, that - at an arbitrary point P - is characterised by *the density of the flow of g-information* at that point. That vectoral quantity can be identified with $\vec{E}_g$, the gravitational field strength, and the cloud of informatons with the gravitational field in ***O***.

A second consequence is that the informatons emitted by a point mass that is moving relative to ***O***, constitute a gravitational field in ***O*** that is characterised by two vectoral quantities: $\vec{E}_g$, the density of the g-information flow and $\vec{B}_g$, *the density of the β-information cloud*. We will show that the relations between these two quantities (the laws of GEM) - the macroscopic expressions of the kinematics of the informatons - are the gravitational analogues of Maxwell's electromagnetic laws.

Next we explain the gravitational interaction between masses as the response of an object to the disturbance of the symmetry of its "proper" gravitational field by the field that, in its direct vicinity, is created and maintained by other masses. And finally we examine the emission of energy by an accelerating mass.

The starting point of GEM and of the theory of informatons differs fundamentally from the starting point of GRT, because space and time don't play an active role neither in the description of gravity by GEM nor in the explanation of the gravitational phenomena and laws by the theory of informatons. In those contexts space and time are elements of the description of nature that do not participate in the physical processes. We still mention that GEM has been discussed within the framework of GRT by a number of authors[7],[8]. They came to the conclusion that the gravitational analogues to Maxwell's equations (the GEM equations) are valid in the weak field approximation.

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CHAPTER 2

THE POSTULATE OF THE EMISSION OF INFORMATONS

The "theory of informatons" explains the gravitational (and the electromagnetic) interactions and phenomena by the hypothesis that "*information*" is the substance of gravitational (and of electromagnetic) fields.

The constituent element of that substance is called an "*informaton*". The theory starts from the idea that any material object manifests itself in space by the continuous emission - at a rate proportional to its rest mass - of informatons: granular mass and energy less entities rushing away with the speed of light and carrying information about the position ("*g-information*") and about the velocity ("*β-information*") of their emitter.

In this chapter the mechanism of the emission of informatons by a point mass at rest will be described, and the informaton will be defined by its attributes.

2.1 PRELIMINARY DEFINITIONS

A material body occupies space, its surface encloses matter. The amount of matter within its contours is called its *mass.* According to the field theory, any material body is the source of a gravitational field that at a sufficiently large distance is independent of the form of the body. This "far field" can be calculated by reducing the body to a mathematical point in which all the mass is accumulated. Such a point is called a "*particle*" or a "*point mass*" and it will be graphically represented by a little sphere. If we can calculate the gravitational field generated by a point mass, integral calculus delivers the methods to calculate the gravitational field generated by any material body. This justifies the fact that we in the first instance focus on the emission of informatons by a point mass.

The phenomena that are the subject of this book are situated in spacetime: they are located in "space" and dated in "time".

1. In the context of the theory of informatons *space* is conceived as a three-dimensional, homogeneous, isotropic, unlimited and empty continuum. This continuum is called the "Euclidean space" because that what there geometrically is possible is determined by the Euclidean geometry. By anchoring a standardized

Cartesian coordinate system to a reference body, an observer can - relative to that reference body - localize each point by three coordinates *x, y, z.*

2. In the same context we define *time* as the monotonically increasing real quantity *t* that is generated by a standard clock•. In a Cartesian coordinate system a standard clock links to each event a "moment" - this is a specific value of *t* - and to each duration a "period" or "time interval" - this is a specific increase of *t.* The introduction of time makes it possible for the observer to express, in an objective manner, the chronological order of events in a Cartesian coordinate system.

A Cartesian coordinate system together with a standard clock is called a "*reference frame*". We represent a reference frame as *OXYZ*(*T*) or shortly as ***O***. A reference frame is called an "*inertial reference frame*" if light propagates rectilinear (in the sense of the Euclidean geometry) with constant speed everywhere in the empty space linked to that frame. This definition implies that the space linked to an inertial reference frame is an homogeneous, isotropic, unlimited and empty continuum in which the Euclidean geometry is valid. A reference frame ***O'*** moving relative to an inertial reference frame ***O*** is itself also an inertial reference frame. The coordinates of an event linked to the inertial frames ***O*** and ***O'*** are related by the Lorentz transformation.

2.2 THE CONCEPT OF GRAVITATIONAL INFORMATION

Newton's law of universal gravitation[1] may be expressed as follows:

The force between any two particles having masses m_1 and m_2 separated by a distance r is an attraction working along the line joining the particles and has a magnitude

$$F = G.\frac{m_1.m_2}{r^2}$$

where G is a universal constant having the same value for all pairs of particles.

This law expresses the basic fact of gravitation, namely that two masses are interacting "at-a-distance": they exert forces on one another even though they are not in contact.

According to Newton's law $\vec{F}_B$, the force exerted by a particle *A* - with mass m_1 - on a particle *B* - with mass *m* - is pointing to the position of *A* and has a magnitude:

• The operation of a standard clock is based on the counting of the successive cycles of a periodic process that is generated by a device inside the clock.

$$F_B = \left(G.\frac{m_1}{r^2}\right).m$$

The orientation of this force and the fact that it is directly proportional to the mass of *A* and inversely proportional to the square of the distance from *A* to *B*, implies that particle *B* must receive *information* about the presence in space of particle *A*: particle *A* must send information to *B* about its position and about its mass. This conclusion is independent of the position and the mass of *B*; so we can generalize it and posit that

A particle manifests itself in space by emitting information about its mass and about its position.

We consider that type of information as a substantial element of nature and call it "*gravitational information*" or "*g-information*". We assume that g-information is transported by mass and energy less granular entities that rush through space with the speed of light (*c*). These grains of g-information are called *informatons*.

2.3 THE POSTULATE OF THE EMISSION OF INFORMATONS

A material object manifests its presence in space by continuously emitting informatons. The emission of informatons by a material object anchored in an inertial reference frame ***O***, is governed by the "*postulate of the emission of informatons*".

A. *The emission* of informatons by a particle at rest is governed by the following rules:

1. The emission is uniform in all directions of space, and the informatons diverge with the speed of light (c = 3.10^8 m/s*) along radial trajectories relative to the position of the emitter.*

2. $\dot{N} = \frac{dN}{dt}$, *the rate at which a particle emits informatons*[•], *is time independent and proportional to the rest mass* m_0 *of the emitter. So there is a constant K so that:*

$$\dot{N} = K.m_0$$

3. The constant K is equal to the ratio of the square of the speed of light (c) to the Planck constant (h):

[•] We neglect the possible stochastic nature of the emission, that is responsible for noise on the quantities that characterize the gravitational field. So, $\dot{N}$ is the average emission rate.

$$K = \frac{c^2}{h} = 1{,}36.10^{50} kg^{-1}.s^{-1}$$

B. We call the essential attribute of an informaton its *g-index.* The g-index of an informaton refers to information about the position of its emitter and equals the *elementary quantity of g-information*. It is represented by a vectoral quantity $\vec{s}_g$:

1. $\vec{s}_g$ *points to the position of the emitter.*

2. The elementary quantity of g-information is:

$$s_g = \frac{1}{K.\eta_0} = 6{,}18.10^{-60} m^3.s^{-1}$$

where $\eta_0 = \frac{1}{4.\pi.G} = 1{,}19.10^9 kg.s^2.m^{-3}$, *G* being the gravitational constant.

Rule A.1 is the expression of the hypothesis that the space is an homogenous and isotropic continuum in which the gravitational phenomena are travelling with the speed of light. Rule A.2 posits that the rate at which a particle emits informatons is a measure for its rest mass and rule A.3 implies the fact that, when a particle absorbs (emits) a photon $h.\nu$, its rest mass is increasing (decreasing) with an amount $\frac{h.\nu}{c^2}$ while its emission rate is increasing (decreasing) with an amount ν. Rule B.1 and rule B.2 respectively express the facts that the gravitational field of a particle always points to the position of the source of that field and that the gravitational force between any two particles depends on a universal constant *G*.

To summarize, each material object manifests itself in space by the emission of informatons, it is a source informatons. Informatons are grains of g-information and, as such, the constituent elements of gravitational fields. In the context of the postulate of the emission of informatons they are completely defined by their g-index $\vec{s}_g$. We will *represent* an informaton as a quasi-infinitely small spinning sphere, moving with velocity $\vec{c}$ and carrying a vector $\vec{s}_g$.

In what follows we will show that informatons macroscopically manifest themselves in $\vec{E}_g$ and $\vec{B}_g$, the vectoral quantities that mathematically characterize gravitational fields; and in the laws of GEM that are manifestations of their kinematics. We will also show that informatons emitted by an accelerated point mass can be carriers of a quantum of energy. The combination of an informaton with a packet of energy appears to the observer as a “graviton”.

It also is possible to explain electromagnetism by the theory of informatons[2],[3]. In that context they macroscopically manifest themselves as $\vec{E}$ and $\vec{B}$, the vectoral quantities that characterize an EM field, and in Maxwell's laws that are manifestations of their kinematics. In the context of EM a "photon" can be interpreted as a combination of an information (the carrier) and a quantum of energy.

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CHAPTER 3

THE GRAVITATIONAL FIELD OF AN OBJECT AT REST

In what follows we will show that the emission of informatons by an object at rest macroscopically manifests itself in the *gravitational field of that object*. The substance of that gravitational field is *g-information.* The gravitational field of an object at rest is completely characterized by a vectoral quantity $\vec{E}_g$, called the *g-field*. $\vec{E}_g$ has a value at every point of space and is thus - relative to an inertial reference frame ***O*** - regarded as a function of the space coordinates. At a certain point *P*, $\vec{E}_g$ is *the density of the g-information flow* passing near *P*. The relation between $\vec{E}_g$ and the rest mass of its source (i.e. the first equation of Maxwell-Heaviside) is the expression of the law of *conservation of g-information*.

3.1 THE GRAVITATIONAL FIELD OF A PARTICLE AT REST

In fig 1 we consider a particle or "*point mass*" with rest mass m_0 that is anchored at the origin of an inertial reference frame ***O***. According to the postulate it continuously emits informatons in all directions of space.

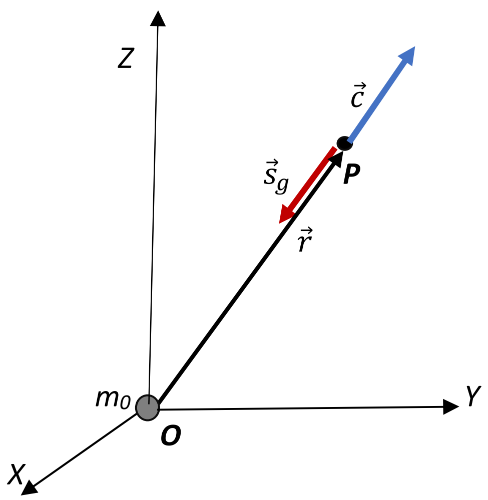

Fig 1

The informatons that with velocity

$$\vec{c} = c.\frac{\vec{r}}{r} = c.\vec{e}_r$$

pass near a fixed point P - defined by the position vector $\vec{r}$ - are characterised by their g-index $\vec{s}_g$:

$$\vec{s}_g = -\frac{1}{K.\eta_0}.\frac{\vec{r}}{r} = -\frac{1}{K.\eta_0}.\vec{e}_r$$

The rate at which the point mass emits g-information is the product of the rate at which it emits informatons with the elementary g-information quantity:

$$\dot{N}.s_g = \frac{m_0}{\eta_0}$$

Of course, this is also the rate at which it sends g-information through any closed surface that surrounds m_0: it is the *intensity of the g-information-flow* through any closed surface that encloses m_0.

The emission of informatons fills the space around m_0 with an expanding cloud of g-information. This cloud has the shape of a sphere whose surface moves away with the speed of light from the centre *O*, the position of the point mass.

1. Within that cloud there is a *stationary state*. Because for each spatial region, the inflow of g-information equals the outflow, each spatial region contains an unchanging number of informatons and thus a constant quantity of g-information. Moreover, the orientation of the g-indices of the informatons passing near a fixed point is always the same.

2. That cloud can be identified with a *continuum*. Each spatial region contains a very large number of informatons: the g-information is like continuously spread over the volume of the region.

The cloud of g-information surrounding O can be identified as the gravitational field or the g-field of the point mass m_0.

Without interruption “countless” informatons are rushing through any - even a very small - surface in the gravitational field: we can describe the motion of g-information through a surface as a *continuous flow* of g-information.

We know already that the intensity of the flow of g-information through a closed surface *that surrounds O is expressed as:*

$$\dot{N}.s_g = \frac{m_0}{\eta_0}$$

If the closed surface is a sphere with radius *r*, the *intensity of the flow per unit area* is given by:

$$\frac{m_0}{4.\pi.r^2.\eta_0}$$

This is the *density of the flow of g-information* at each point *P* at a distance *r* from m_0 (fig 1). This quantity is, together with the orientation of the g-indices of the informatons that are passing near *P*, characteristic for the gravitational field at that point. Thus, at a point *P*, the gravitational field of the point mass m_0 is unambiguously defined by the vectoral quantity $\vec{E}_g$:

$$\vec{E}_g = \frac{\dot{N}}{4.\pi.r^2}.\vec{s}_g = -\frac{m_0}{4.\pi.\eta_0.r^2}.\vec{e}_r = -\frac{m_0}{4.\pi.\eta_0.r^3}.\vec{r}$$

This quantity is the *gravitational field strength* or the *g-field strength* or the *g-field*. In any point of the gravitational field of the point mass m_0, the orientation of $\vec{E}_g$ corresponds to the orientation of the g-indices of the informatons which are passing near that point. And the magnitude of $\vec{E}_g$ is the *density of the g-information flow* at that point. Let us note that $\vec{E}_g$ is opposite to the sense of movement of the informatons.

Finally, let us consider a surface-element *dS* at *P* (fig 2,a). Its orientation and magnitude are completely determined by the surface-vector $\overrightarrow{dS}$ (fig 2,b). By $-d\Phi_G$, we represent the rate at which g-information flows through *dS* in the sense of the positive normal $\vec{e}_n$ and we call the scalar quantity $d\Phi_G$ the *elementary g-flux through dS*:

$$d\Phi_G = \vec{E}_g.\overrightarrow{dS} = E_g.dS.\cos\alpha$$

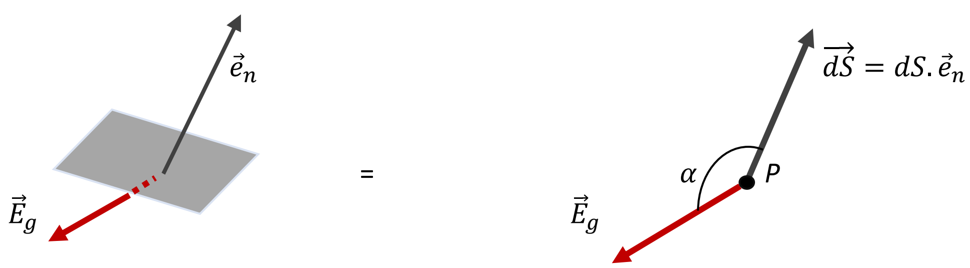

Fig 2,a

Fig 2,b

For an arbitrary closed surface *S* that surrounds m_0, the outward flux (which we obtain by integrating the elementary contributions $d\Phi_g$ over *S*) must be equal to the rate at which the mass emits g-information. Thus:

$$\Phi_G = \oiint \vec{E}_g \,.\, \overrightarrow{dS} = -\frac{m_0}{\eta_0}$$

This relation is the expression of the conservation of g-information in the case of a point mass at rest.

3.2 THE GRAVITATIONAL FIELD OF A SET OF PARTICLES AT REST

We consider a set of particles with rest masses $m_1,\ldots,m_i,\ldots m_n$ that are anchored in an inertial reference frame ***O***. At an arbitrary point *P*, the flows of g-information who are emitted by the distinct masses are defined by the gravitational fields $\vec{E}_{g1},\ldots,\vec{E}_{gi},\ldots,\vec{E}_{gn}$. $-d\Phi_g$, the rate at which g-information flows through a surface-element *dS* at *P* in the sense of the positive normal, is the sum of the contributions of the distinct masses:

$$-d\Phi_G = \sum_{i=1}^{n} -(\vec{E}_{gi} \,.\, \overrightarrow{dS}) = -(\sum_{i=1}^{n} \vec{E}_{gi}).\overrightarrow{dS} = -\vec{E}_g.\overrightarrow{dS}$$

So, the *effective density of the flow of g-information at P* (the effective g-field) is completely defined by:

$$\vec{E}_g = \sum_{i=1}^{n} \vec{E}_{gi}$$

We conclude:

At a point of space, the g-field of a set of point masses at rest is completely defined by the vectoral sum of the g-fields caused by the distinct masses.

Let us remark that the orientation of the effective g-field has no longer a relation with the direction in which the passing informatons are moving.

One easily shows that the outward g-flux through a closed surface in the g-field of a set of anchored point masses only depends on the surrounded masses m_{in}:

$$-\oiint \vec{E}_g \,.\, \overrightarrow{dS} = \frac{m_{in}}{\eta_0}$$

This relation is the expression of the conservation of g-information in the case of a set of point masses at rest.

3.3 THE GRAVITATIONAL FIELD OF A MASS CONTINUUM AT REST

We call an object in which the matter in a time independent manner is spread over the occupied volume, a *mass continuum*. At each point Q of such a continuum, the accumulation of mass is defined by the (*mass*) *density* ρ_G. To define this scalar quantity one considers the mass *dm* of a volume element *dV* that contains Q. The accumulation of mass in the vicinity of Q is defined by:

$$\rho_G = \frac{dm}{dV}$$

A mass continuum - anchored in an inertial reference frame - is equivalent to a set of infinitely many infinitesimal small mass elements *dm*. The contribution of each of them to the field strength at an arbitrary point P is $d\vec{E}_g$. $\vec{E}_g$, the effective g-field at P, is the result of the integration over the volume of the continuum of all these contributions.

It is evident that the outward g-flux through a closed surface S only depends on the mass enclosed by that surface (the enclosed volume is V):

$$-\oiint_S \vec{E}_g \, . \overrightarrow{dS} = \frac{1}{\eta_0} . \iiint_V \rho_G . dV$$

That relation is equivalent with (theorem of Ostrogradsky[2]):

$$div\vec{E}_g = -\frac{\rho_G}{\eta_0}$$

This is the expression of the conservation of g-information in the case of a mass continuum at rest.

Furthermore, one can show that[2],[3] $rot\vec{E}_g = 0$, what implies the existence of a gravitational potential function V_g for which:

$$\vec{E}_g = -gradV_g$$

3.4 CONCLUSION

The gravitational field of a particle at rest forms an indivisible whole with that particle. It is completely characterized by the physical quantity “gravitational field” or “g-field”. This quantity is represented by the position dependent vector $\vec{E}_g$, the density of the flow of g-information at an arbitrary point P.

The substance of the gravitational field is “g-information” and its constituent element is the “informaton”. This implies that the gravitational field is granular, that it continuously regenerates, that it shows fluctuations, that it expands with the speed of light, that gravitational phenomena propagate with that speed and that there is conservation of g-information at every point of the gravitational field.

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CHAPTER 4

THE GRAVITATIONAL FIELD OF AN OBJECT MOVING WITH CONSTANT VELOCITY

To characterize the gravitational field of a moving object we need a vector field with two components: the *g-field* $\vec{E}_{\boldsymbol{g}}$ and the *g-induction* $\vec{B}_{\boldsymbol{g}}$ that respectively define the density of the flow of g-information and the density of the cloud of β-information at every point of space and time. We show that the gravitational field of an object moving with constant velocity is governed by the Maxwell-Heaviside equations and that these equations in no way lead to the conclusion that there are causal relations between the changes in time and the spatial variations of $\vec{E}_{\boldsymbol{g}}$ and $\vec{B}_{\boldsymbol{g}}$. The gravitational field is a dual entity having a field and an induction component.

4.1 THE g-FIELD OF A PARTICLE MOVING WITH CONSTANT VELOCITY

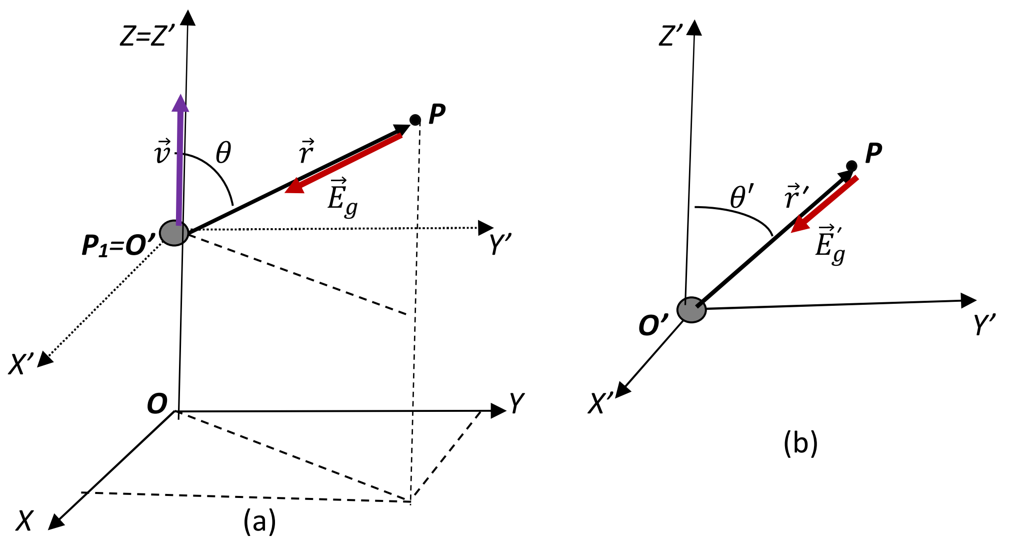

Fig 3

In fig 3,a, we consider a particle with rest mass m_0 that is moving with constant velocity $\vec{v} = v.\vec{e}_z$ along the *Z*-axis of an IRF ***O***. At the moment $t = 0$**,** it passes through the origin *O* and at the moment $t = t$ through the point P_1. It is evident that:

$$OP_1 = z_{P_1} = v.t$$

P is an arbitrary fixed point in ***O*** with space coordinates (x, y, z). . Its position relative to the moving particle is determined by the time dependent position vector $\vec{r} = \overrightarrow{P_1P}$.

The g-field at P is the vectoral quantity $\vec{E}_g$ that at that point characterizes the density of the flow of g-information. The magnitude of $\vec{E}_g$ is the rate per unit area at which g-information at P flows through an elementary surface perpendicular to the direction of $\vec{E}_g$.

We introduce the IRF ***O'*** (fig 3,a) whose origin is anchored to the moving particle and we assume that $t = t' = 0$ when it passes through O.

Relative to ***O'*** where the particle is at rest at the origin O' (fig 3,b), the position of the point P is determined by the time dependent position vector $\vec{r}' = \overrightarrow{O'P}$ so that in ***O'*** the space coordinates of P are (x', y', z').

Because the particle is at rest in ***O'***, $\vec{E}'_g$ - the density of the g-information flow at P relative to ***O'*** is – according to §3.1 defined by the vectoral quantity:

$$\overrightarrow{E'}_g = -\frac{m_0}{4.\pi.\eta_0.r'^2}.\vec{e}_{r'} = -\frac{m_0}{4\pi\eta_0 r'^3}.\overrightarrow{r'}$$

The components of $\vec{E}'_g$ in ***O'***, are:

$$E'_{gx'} = -\frac{m_0}{4\pi\eta_0 r'^3}.x'$$

$$E'_{gy'} = -\frac{m_0}{4\pi\eta_0 r'^3}.y'$$

$$E'_{gz'} = -\frac{m_0}{4\pi\eta_0 r'^3}.z'$$

They determine at P the densities of the flows of g-information respectively through a surface element $dy'.dz'$ perpendicular to the X'-axis, through a surface element $dz'.dx'$ perpendicular to the Y'-axis and through a surface element $dx'.dy'$ perpendicular to the Z'-axis. Thus, the rates at which g-information is flowing through these different surface elements (the g-fluxes) at P are:

$$E'_{gx'}.dy'.dz' = -\frac{m_0.x'}{4\pi\eta_0 r'^3}.dy'.dz'$$

$$E'_{gy'}.dz'.dx' = -\frac{m_0.y'}{4\pi\eta_0 r'^3}.dz'.dx'$$

$$E'_{gz'}.dx'.dy' = -\frac{m_0.z'}{4\pi\eta_0 r'^3}.dx'.dy'$$

Informatons propagate at the speed of light that - in free space - has the same value in all inertial reference frames. That implies that the rate at which g-information flows through a surface element *dS* in ***O*** can be derived from the rate at which it flows through a surface element *dS'* in ***O'*** by applicating the Lorenz transformation equations.

- The Cartesian coordinates of *P* in the frames ***O*** and ***O'*** are related to each other by[1]:

  x' = x *y' =y* $z' = \frac{z-v.t}{\sqrt{1-\beta^2}} = \frac{z-z_{P_1}}{\sqrt{1-\beta^2}}$

- The line elements by: *dx' =dx* *dy'=dy* $dz' = \frac{dz}{\sqrt{1-\beta^2}}$
- And further:

$$r' = r.\frac{\sqrt{1-\beta^2.sin^2\theta}}{\sqrt{1-\beta^2}}$$

So relative to ***O***, the rates at which the moving particle sends g-information in the positive direction through the surface elements *dy.dz*, *dz.dx* and *dx.dy* at *P* are:

$$-\frac{m_0}{4\pi\eta_0 r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\theta)^{\frac{3}{2}}}.x.dy.dz$$
$$-\frac{m_0}{4\pi\eta_0 r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\theta)^{\frac{3}{2}}}.y.dx.dz$$

$$-\frac{m_0}{4\pi\eta_0 r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\theta)^{\frac{3}{2}}}.\left(z-z_{P_1}\right).dx.dz$$

By definition, the densities at *P* of the flows of g-information in the direction of the *X*-, the *Y*- and the *Z*-axis are the components of the g-field caused by the moving particle m_0 at *P* in ***O***. So:

$$E_{gx} = -\frac{m_0}{4\pi\eta_0 r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\theta)^{\frac{3}{2}}}.x$$

$$E_{gy} = -\frac{m_0}{4\pi\eta_0 r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\theta)^{\frac{3}{2}}}.y$$

$$E_{gz} = -\frac{m_0}{4\pi\eta_0 r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\theta)^{\frac{3}{2}}}.(z - z_{P_1})$$

From which it follows that the g-field caused by the particle at the fixed point P is:

$$\vec{E}_g = -\frac{m_0}{4\pi\eta_0 r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\theta)^{\frac{3}{2}}}.\vec{r} = -\frac{m_0}{4\pi\eta_0 r^2}.\frac{1-\beta^2}{(1-\beta^2.sin^2\theta)^{\frac{3}{2}}}.\vec{e}_r$$

We conclude:

> *A particle describing a uniform rectilinear movement relative to an inertial reference frame* ***O****, creates in the space linked to that frame a time dependent gravitational field.* $\vec{E}_g$*, the g-field at an arbitrary point P, points at any time to the position of the mass at that moment*• *and its magnitude is:*

$$E_g = \frac{m_0}{4\pi\eta_0 r^2}.\frac{1-\beta^2}{(1-\beta^2.sin^2\theta)^{\frac{3}{2}}} \qquad (1)$$

In §3.1 we concluded that for an arbitrary closed surface S that surrounds m_0, a particle at rest relative to an IRF ***O***, the outward flux must be equal to the rate at which the mass emits g-information. Thus:

$$\Phi_G = \oiint \vec{E}_g\,.\,\overrightarrow{dS} = -\frac{m_0}{\eta_0}$$

• The orientation of the g-field implies that the g-indices of the informatons that at a certain moment pass near P, point to the position of the emitting mass at that moment and not to its light delayed position.

> *This relation, the expression of conservation of g-information, applies also in the case of a particle m_0 moving with constant velocity relative to **O**.*

Indeed, the relation applies in the proper IRF ***O'*** of m_0 (the IRF anchored to m_0) and because informatons (the carriers of g-information) travel with the same speed in all inertial reference frames, the rate at which g-information flows through a surface element *dS* doesn't depend on the IRF relative to which *dS* is described.

If the speed of the mass is much smaller than the speed of light, the expression (1) reduces to that valid in the case of a mass at rest. This non-relativistic result could directly be obtained if one assumes that the displacement of the point mass during the time interval that the informatons need to move from the emitter to *P* can be neglected compared to the distance they travel during that period.

4.2 THE EMISSION OF INFORMATONS BY A PARTICLE MOVING WITH CONSTANT VELOCITY

In fig 4 we consider a particle with rest mass m_0 that is moving with constant velocity $\vec{v}$ along the *Z*-axis of an inertial reference frame ***O***. Its instantaneous position (at the arbitrary moment *t*) is P_1. The position of *P*, an arbitrary fixed point in space, is defined by the vector $\vec{r} = \overrightarrow{P_1 P}$. This position vector $\vec{r}$ - just like the distance *r* and the angle θ - is time dependent because the position of P_1 is constantly changing.

The informatons that - with the speed of light - at the moment *t* are passing near *P*, are emitted when m_0 was at P_0. Bridging the distance $P_0 P = r_0$ took the time interval $\Delta t = \frac{r_0}{c}$.

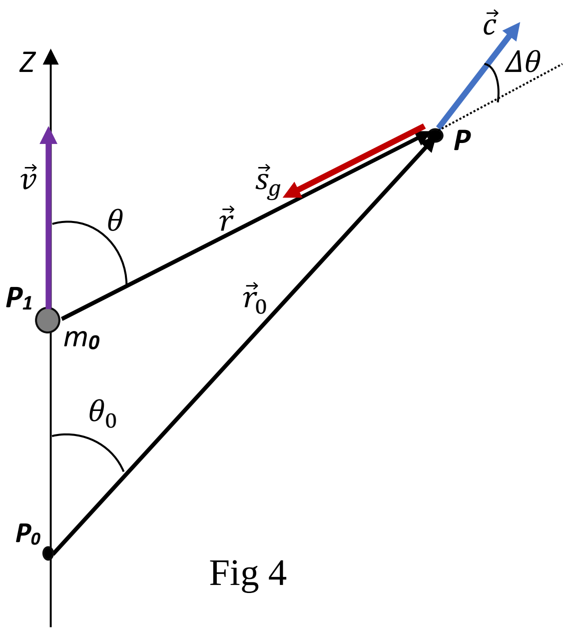

Fig 4

During their rush from P_0 to P their emitter, the particle, moved from P_0 to P_1: $P_0P_1 = v.\Delta t$

1. $\vec{c}$, the velocity of the informatons, points in the direction of their movement, thus along the radius P_0P;

2. $\vec{s}_g$, their g-index, points to P_1, the position of m_0 at the moment t. This is an implication of rule B.1 of the postulate of the emission of informatons, confirmed by the conclusion of §4.2.

The lines carrying $\vec{s}_g$ and $\vec{c}$ form an angle $\Delta\theta$. We call this angle - that is characteristic for the speed of the point mass - the "*characteristic angle*" or the "*characteristic deviation*". The quantity $s_\beta = s_g.sin(\Delta\theta)$, referring to the speed of its emitter, is called the "*characteristic g-information*" or the "*β -information*" *o*f an informaton.

We conclude that an informaton emitted by a moving particle, transports information referring to the velocity of that particle. This information is represented by its "*gravitational characteristic vector*" or its "*β -index*" $\vec{s}_\beta$ that is defined by:

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c}$$

- The β-index is perpendicular to the plane formed by the path of the informaton and the straight line that carries the g-index, thus it is perpendicular to the plane formed by the point P and the path of the emitter.

- Its orientation relative to that plane is defined by the "rule of the corkscrew".

- Its magnitude is: $\mathrm{s}_\beta = \mathrm{s}_\mathrm{g}.\sin(\Delta\theta)$, the β -information of the informaton.

In the case of fig 4 the β-indices have the orientation of the positive *X*-axis.

Applying the sine-rule to the triangle P_0P_1P, we obtain:

$$\frac{sin(\Delta\theta)}{v.\Delta t} = \frac{sin\,\theta}{c.\Delta t}$$

From which it follows:

$$s_\beta = s_g.\frac{v}{c}.\sin\theta = s_g.\beta.\sin\theta = s_g.\beta_\perp$$

$\beta_\perp$ is the component of the dimensionless velocity $\vec{\beta} = \frac{\vec{v}}{c}$ perpendicular to $\vec{s}_g$

Taking into account the orientation of the different vectors, the β-index of an informaton emitted by a point mass moving with constant velocity, can also be expressed as:

$$\vec{s}_\beta = \frac{\vec{v} \times \vec{s}_g}{c}$$

4.3 THE GRAVITIONAL INDUCTION OF A PARTICLE MOVING WITH CONSTANT VELOCITY

We consider again the situation of fig 3. All informatons in *dV* - the volume element at *P* - carry both g-information and β-information. The β-information refers to the velocity of the emitting mass and is represented by the β-indices $\vec{s}_\beta$:

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c} = \frac{\vec{v} \times \vec{s}_g}{c}$$

If *n* is the density at *P* of the cloud of informatons (number of informatons per unit volume) at the moment *t*, the amount of β-information in *dV* is determined by the magnitude of the vector:

$$n.\vec{s}_\beta.dV = n.\frac{\vec{c} \times \vec{s}_g}{c}.dV = n.\frac{\vec{v} \times \vec{s}_g}{c}.dV$$

And the density of the β-information (characteristic information per unit volume) at *P* is determined by:

$$n.\vec{s}_\beta = n.\frac{\vec{c} \times \vec{s}_g}{c} = n.\frac{\vec{v} \times \vec{s}_g}{c}$$

We call this (time dependent) vectoral quantity - that will be represented by $\vec{B}_g$ - the "*gravitational induction*" or the "*g-induction*" at *P*•:

• also called "*gravitomagnetic induction*"

- Its magnitude B_g determines the density of the β-information at *P;*

- Its orientation determines the orientation of the β-indices $\vec{s}_\beta$ of the informatons passing near that point.

So, the g-induction caused by the moving mass m_0 (fig 3) at *P* is:

$$\vec{B}_g = n.\frac{\vec{v} \times \vec{s}_g}{c} = \frac{\vec{v}}{c} \times (n.\vec{s}_g)$$

N - the density of the flow of informatons at *P* (the rate per unit area at which the informatons cross an elementary surface perpendicular to the direction of movement) - and *n* - the density of the cloud of informatons at *P* (number of informatons per unit volume) - are connected by the relation:

$$n = \frac{N}{c}$$

With $\vec{E}_g = N.\vec{s}_g$, we can express the gravitational induction at *P* as:

$$\vec{B}_g = \frac{\vec{v}}{c^2} \times (N.\vec{s}_g) = \frac{\vec{v} \times \vec{E}_g}{c^2}$$

Taking the result of §4.2 into account, namely:

$$\vec{E}_g = -\frac{m_0}{4\pi\eta_0 r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\,\theta)^{\frac{3}{2}}}.\vec{r}$$

We find:

$$\vec{B}_g = -\frac{m_0}{4\pi\eta_0 c^2.r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\,\theta)^{\frac{3}{2}}}.(\vec{v} \times \vec{r})$$

We define the constant ν_0 = *9,34.10*$^{-27}$ *m.kg*$^{-1}$ as:

$$\nu_0 = \frac{1}{c^2.\eta_0}$$

And finally, we obtain:

$$\vec{B}_g = \frac{\nu_0 . m_0}{4\pi r^3} . \frac{1-\beta^2}{(1-\beta^2 . sin^2\,\theta)^{\frac{3}{2}}} . (\vec{r} \times \vec{v})$$

$\vec{B}_g$ at P is perpendicular to the plane formed by P and the path of the point mass; its orientation is defined by the rule of the corkscrew; and its magnitude is:

$$B_g = \frac{\nu_0 . m_0}{4\pi r^2} . \frac{1-\beta^2}{(1-\beta^2 . sin^2\,\theta)^{\frac{3}{2}}} . v . sin\,\theta$$

If the speed of the mass is much smaller than the speed of light, the expression for the gravitational induction reduces itself to:

$$\vec{B}_g = \frac{\nu_0 . m_0}{4\pi r^3} . (\vec{r} \times \vec{v})$$

This non-relativistic result could directly be obtained if one assumes that the displacement of the point mass during the time interval that the informatons need to move from the emitter to P can be neglected compared to the distance they travel during that period. This means that for situations where $v \ll c$, in the previous calculation the formula

$$\vec{E}_g = -\frac{m_0}{4 . \pi . \eta_0 . r^3} . \vec{r}$$

can be used to express the g-field.

So if $v \ll c$, $\vec{B}_g$ at P is perpendicular to the plane formed by P and the path of the point mass; its orientation is defined by the rule of the corkscrew; and its magnitude is:

$$B_g = \frac{\nu_0 . m_0}{4\pi r^2} . v . sin\,\theta$$

4.4 THE GRAVITATIONAL FIELD OF A PARTICLE MOVING WITH CONSTANT VELOCITY

A particle m_0, moving with constant velocity $\vec{v} = v.\vec{e}_z$ along the Z-axis of an inertial reference frame, creates and maintains an expanding cloud of informatons that are carrying both g- and β-information. That cloud can be identified with a time dependent continuum. That continuum is called the *gravitational field*[•] of the point mass. It is characterized by two time dependent vectoral quantities: the gravitational field (short: *g-field*) $\vec{E}_g$ and the gravitational induction (short*: g-induction*) $\vec{B}_g$.

1. With *N* the density of the flow of informatons at *P* (the rate per unit area at which the informatons cross an elementary surface perpendicular to the direction of movement), the g-field at that point is:

$$\vec{E}_g = N.\vec{s}_g = -\frac{m_0}{4\pi\eta_0 r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\,\theta)^{\frac{3}{2}}}.\vec{r}$$

The orientation of $\vec{E}_g$ learns that the direction of the flow of g-information at *P* is not the same as the direction of the flow of informatons.

2. With *n,* the density of the cloud of informatons at *P* (number of informatons per unit volume), the g-induction at that point is:

$$\vec{B}_g = n.\vec{s}_\beta = \frac{\nu_0.m_0}{4\pi r^3}.\frac{1-\beta^2}{(1-\beta^2.sin^2\,\theta)^{\frac{3}{2}}}.(\vec{r}\times\vec{v})$$

One can verify (Appendix A) that:

1. $div\vec{E}_g = 0$ 2. $div\vec{B}_g = 0$

3. $rot\vec{E}_g = -\frac{\partial\vec{B}_g}{\partial t}$ 4. $rot\vec{B}_g = \frac{1}{c^2}.\frac{\partial\vec{E}_g}{\partial t}$

These relations are the laws of GEM (Maxwell-Heaviside) in the case of the gravitational field of a particle describing a uniform rectilinear motion. It is important to notice that (3) and (4) express how the respective changes in space and time are linked to each other, and that (3) and (4) don't express causal relationships. The gravitational field is a dual entity having a field and an induction component.

[•] Also called: "gravito-electromagnetic" (GEM field) or "gravito-magnetic" field (GM field)

If $v << c$, the expressions for the g-field and the g-induction reduce to:

$$\vec{E}_g = -\frac{m_0}{4\pi\eta_0 r^3}.\vec{r} \qquad\qquad \vec{B}_g = \frac{\nu_0.m_0}{4\pi r^3}.(\vec{r}\times\vec{v})$$

4.5 THE GRAVITATIONAL FIELD OF A SET OF PARTICLES MOVING WITH CONSTANT VELOCITIES

We consider a set of particles $m_1,\ldots,m_i,\ldots m_n$ that move with constant velocities $\vec{v}_1,\ldots,\vec{v}_i,\ldots,\vec{v}_n$ relative to an inertial reference frame ***O***. This set creates and maintains a gravitational field that at each point of the space linked to ***O***, is characterised by the vector pair $(\vec{E}_g, \vec{B}_g)$.

1. Each mass m_i continuously emits g-information and contributes with an amount $\vec{E}_{gi}$ to the g-field at an arbitrary point *P*. As in §3.2 we conclude that the effective g-field $\vec{E}_g$ at *P* is defined as:

$$\vec{E}_g = \sum \vec{E}_{gi}$$

2. If it is moving, each mass m_i emits also β-information, contributing to the g- induction at *P* with an amount $\vec{B}_{gi}$. It is evident that the β–information in the volume element *dV* at *P* at each moment *t* is expressed by:

$$\sum(\vec{B}_{gi}.dV) = (\sum \vec{B}_{gi}).dV$$

Thus, the effective g-induction $\vec{B}_g$ at *P* is:

$$\vec{B}_g = \sum \vec{B}_{gi}$$

On the basis of the superposition principle we can conclude that the laws of GEM mentioned in the previous section remain valid for the effective g-field and g-induction in the case of the gravitational field of a set of particles describing uniform rectilinear motions

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4.6 THE GRAVITATIONAL FIELD OF A STATIONARY MASS FLOW

The term "stationary mass flow" refers to the movement of an homogeneous and incompressible fluid that, in an invariable way, flows relative to an inertial reference frame. The intensity of the flow at an arbitrary point *P* is characterized by the flow density $\vec{J}_G$. The magnitude of this vectoral quantity at *P* equals the rate per unit area at which the mass flows through a surface element that is perpendicular to the flow at *P*. The orientation of $\vec{J}_G$ corresponds to the direction of that flow. If $\vec{v}$ is the velocity of the mass element $\rho_G.dV$ that at the moment *t* flows through *P*, then:

$$\vec{J}_G = \rho_G.\vec{v}$$

So, the rate at which the flow transports – in the positive sense (defined by the orientation of the surface vectors $\overrightarrow{dS}$) - mass through an arbitrary surface *ΔS*, is:

$$i_G = \iint_{\Delta S} \vec{J}_G.\overrightarrow{dS}$$

We call i_G the *intensity of the mass flow through ΔS.*

Since a stationary mass flow is the macroscopic manifestation of moving mass elements $\rho_G.dV$, it creates and maintains a gravitational field. And since the velocity $\vec{v}$ of the mass element at a certain point is time independent, *the gravitational field of a stationary mass flow will be time independent*. It is evident that the rules of §3.3 also apply for this time independent g-field:

1. $div\vec{E}_g = -\dfrac{\rho_G}{\eta_0}$

2. $rot\vec{E}_g = 0$ what implies: $\vec{E}_g = -gradV_g$

One can prove[2],[3],[4] that the rules for the time independent g-induction are:

1. $div\vec{B}_g = 0$ what implies the existence of a vector gravitational potential function $\vec{A}_g$ for which $\vec{B}_g = rot\vec{A}_g$

2. $rot\vec{B}_g = -\nu_0.\vec{J}_G$

These are the laws of GEM in the case of the gravitational field of a stationary mass flow.

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CHAPTER 5

THE GRAVITATIONAL FIELD OF AN ACCELERATED OBJECT

An accelerated object is the source of a gravitational field that, at a sufficient great distance *r* from that object, is characterized by a transverse g-field and g-induction that are both inversely proportional to *r*.

5.1 THE g-INDEX OF AN INFORMATON EMITTED BY AN ACCELERATED PARTICLE

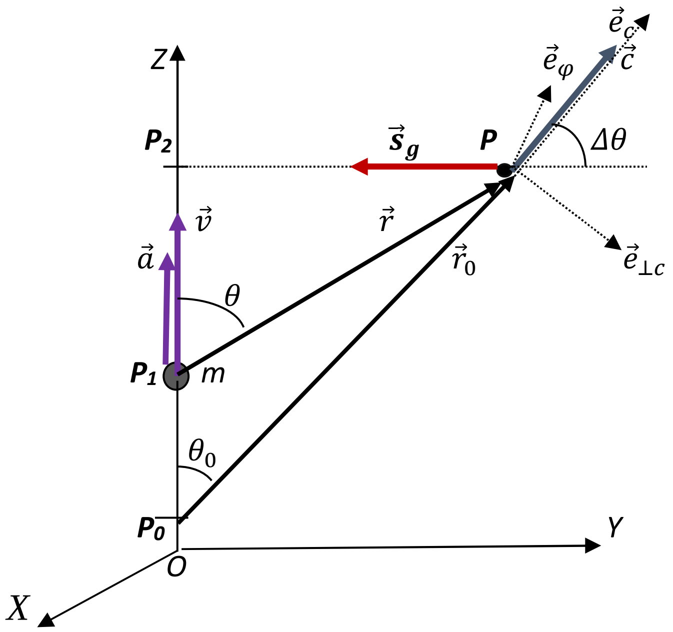

Fig 5

In fig 5 we consider a point mass m that, during a finite time interval, moves with constant acceleration $\vec{a} = a.\vec{e}_z$ relative to the inertial reference frame *OXYZ*. At the moment $t = 0$, m starts from rest at the origin *O*, and at $t = t$ it passes at the point P_1. Its velocity is there defined by $\vec{v} = v.\vec{e}_z = a.t.\vec{e}_z$, and its position by

$$z = \frac{1}{2}.a.t^2 = \frac{1}{2}.v.t$$

We suppose that the speed v remains much smaller than the speed of light: $\frac{v}{c} \ll 1$.

The informatons that during the infinitesimal time interval (t, $t+dt$) pass near the fixed point P (whose position relative to the moving mass m is defined by the time dependent position vector $\vec{r}$) have been emitted at the moment $t_0 = t - \Delta t$, when m – with velocity $\vec{v}_0 = v_0.\vec{e}_z = v(t-\Delta t).\vec{e}_z$ – passed at P_0 (the position of P relative to P_0 is defined by the time dependent position vector $\vec{r}_0 = \vec{r}(t - \Delta t)$). Δt, the time interval during which m moves from P_0 to P_1 is the time that the informatons need to move – with the speed of light – from P_0 to P. We can conclude that $\Delta t = \frac{r_0}{c}$, and that

$$v_0 = v(t - \Delta t) = v(t - \frac{r_0}{c}) = v - a.\frac{r_0}{c}$$

Between the moments $t = t_0$ and $t = t_0 + \Delta t$, m moves from P_0 to P_1. That movement can be considered as the resultant (the superposition) of

1. a uniform movement with constant speed $v_0 = v(t - \Delta t)$ and
2. a uniformly accelerated movement with constant acceleration a.

1.

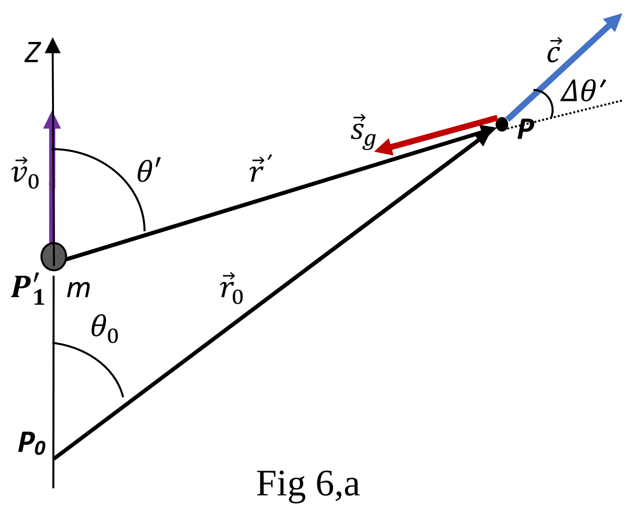

Fig 6,a

In fig. 6,a, we consider the case of the point mass m moving with constant speed v_0 along the Z-axis. At the moment $t_0 = t - \Delta t$ m passes at P_0 and at the moment t at P_1': $P_0P_1' = v_0.\Delta t$. The informatons that, during the infinitesimal time interval (t, $t + dt$), pass near the point P - whose position relative to the uniformly moving mass m at the moment t is defined by the position vector $\vec{r}'$ -

have been emitted at the moment t_0 when m passed at P_0. Their velocity vector $\vec{c}$ is on the line P_0P, their g-index $\vec{s}_g$ points to P_1':

$$P_0P_1' = v_0.\Delta t = v_0\frac{r_0}{c}$$

2. In fig 6,b we consider the case of the point mass m starting at rest at P_0 and moving with constant acceleration a along the Z-axis.

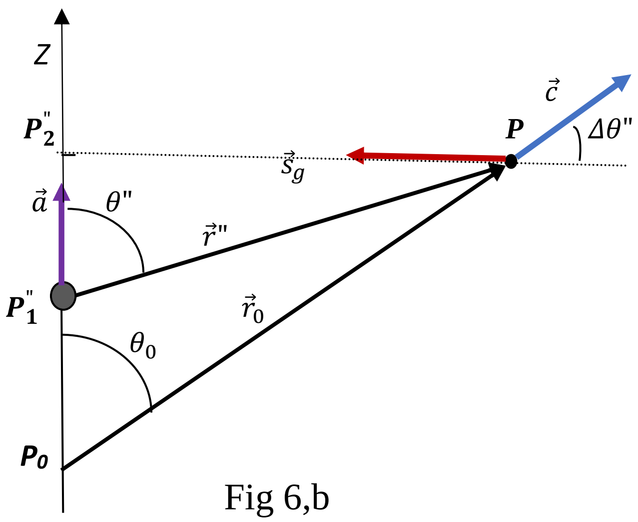

Fig 6,b

At the moment $t_0 = t - \Delta t$ it is at P_0 and at the moment t at P_1'':

$$P_0P_1'' = \frac{1}{2}.a.(\Delta t)^2$$

The informatons that during the infinitesimal time interval (t, $t + dt$) pass near the point P (whose position relative to the uniformly accelerated mass m is at t defined by the position vector $\vec{r}''$) have been emitted at t_0 when m was at P_0. Their velocity vector $\vec{c}$ points away from P_0 , their g-index $\vec{s}_g$ to P_2''.

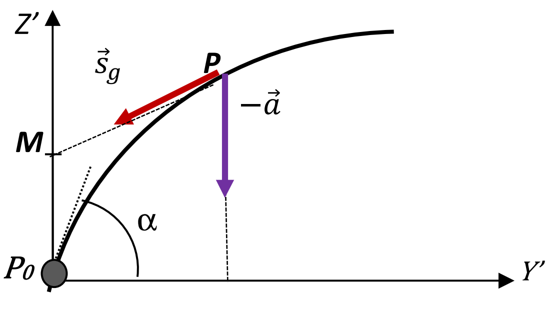

Fig. 6,c

To determine the position of $P_2^{"}$, we consider the trajectory of the informatons that at t_0 are emitted in the direction of P relative to the accelerated reference frame *OX'Y'Z'* that is anchored to *m* (fig 6,c; $\alpha = \frac{\pi}{2} - \theta_0$).

Relative to *OX'Y'Z'* these informatons are accelerated with an amount $-\vec{a}$: they follow a parabolic trajectory described by the equation:

$$z' = tg\alpha . y' - \frac{1}{2} . \frac{a}{c^2 . cos^2 \alpha} . y'^2$$

At the moment $t = t_0 + \Delta t$, when they pass at *P*, the tangent line to that trajectory cuts the *Z'*-axis at the point *M*, that is defined by:

$$z'_M = \frac{1}{2} . a . (\Delta t)^2 = \frac{1}{2} . a . \frac{r_0^2}{c^2}$$

That means that the g-indices of the informatons that at the moment *t* pass at *P*, point to a point *M* on the *Z*-axis that has a lead of

$$P_1^{"} P_2^{"} = P_0 M = \frac{1}{2} . a . (\Delta t)^2 = \frac{1}{2} . a . \frac{r_0^2}{c^2}$$

on $P_1^{"}$, the actual position of the mass *m*. And since $P_0 P_1^{"} = P_0 P_1^{"} + P_1^{"} P_2^{"}$, we conclude that:

$$P_0 P_2^{"} = a . \frac{r_0^2}{c^2}$$

In the inertial reference frame *OXYZ* (fig 6), $\vec{s}_g$ points to the point P_2 on the *Z*-axis determined by the superposition of the effect of the velocity (*1*) and the effect of the acceleration (*2*):

$$P_0 P_2 = P_0 P_1' + P_0 P_2^{"} = \frac{v_0}{c} . r_0 + \frac{a}{c^2} . r_0^2$$

The carrier line of the g-index $\vec{s}_g$ of an informaton that - relative to the inertial frame *OXYZ* - at the moment *t* passes near *P* forms a "characteristic angle" $\Delta\theta$ with the carrier line of its velocity vector $\vec{c}$, that can be deduced by application of the sine-rule in triangle $P_0 P_2 P$ (fig 5):

$$\frac{sin(\Delta\theta)}{P_0P_2} = \frac{sin(\theta_0 + \Delta\theta)}{r_0}$$

We conclude:

$$sin(\Delta\theta) = \frac{v_0}{c}.sin(\theta_0 + \Delta\theta) + \frac{a}{c^2}.r_0.sin(\theta_0 + \Delta\theta)$$

From the fact that P_0P_1 - the distance travelled by *m* during the time interval Δt - can be neglected relative to P_0P - the distance travelled by light during the same period - it follows that $\theta_0 \approx \theta_0 + \Delta\theta \approx \theta$ and that $r_0 \approx r$. So:

$$sin(\Delta\theta) \approx \frac{v_0}{c}.sin\,\theta + \frac{a}{c^2}.r.sin\,\theta$$

We can conclude that the g-index $\vec{s}_g$ of an informaton that at the moment *t* passes near *P*, has a longitudinal component, this is a component in the direction of $\vec{c}$ (its velocity vector) and a transversal component, this is a component perpendicular to that direction. It is evident that:

$$\vec{s}_g = -s_g.cos(\Delta\theta).\vec{e}_c - s_g.sin(\Delta\theta).\vec{e}_{\perp c}$$

$$\approx -s_g.\vec{e}_c - s_g.\left(\frac{v_0}{c}.sin\,\theta + \frac{a}{c^2}.r.sin\,\theta\right).\vec{e}_{\perp c}$$

5.2 THE GRAVITATIONAL FIELD OF AN ACCELERATED PARTICLE

The informatons that, at the moment *t*, are passing near the fixed point *P* - defined by the time dependent position vector $\vec{r}$ - are emitted when *m* was at P_0 (fig 6). Their velocity $\vec{c}$ is on the same carrier line as $\vec{r}_0 = \overrightarrow{P_0P}$. Their g-index is on the carrier line P_2P. According to §5.1, the characteristic angle $\Delta\theta$ - this is the angle between the carrier lines of $\vec{s}_g$ and $\vec{c}$ - has two components:

1. a component $\Delta\theta'$ related to the velocity of *m* at the moment $(t - \frac{r_0}{c})$ when the considered informatons were emitted. In the framework of our assumptions, this component is determined by:

$$sin(\Delta\theta') = \frac{v(t - \frac{r}{c})}{c}.sin\,\theta$$

2. a component $\Delta\theta''$ related to the acceleration of *m* at the moment when they were emitted. This component is, in the framework of our assumptions, determined by:

$$sin(\Delta\theta'') = \frac{a(t - \frac{r}{c}).r}{c^2}.sin\,\theta$$

The macroscopic effect of the emission of g-information by the accelerated mass *m* is a gravitational field $(\vec{E}_g, \vec{B}_g)$. We introduce the reference system $(\vec{e}_c, \vec{e}_{\perp c}, \vec{e}_\varphi)$ (fig 6).

1. $\vec{E}_g$, the g-field at *P*, is defined as the density of the flow of g-information at that point. That density is the rate at which g-information crosses per unit area the elementary surface perpendicular to the direction of movement of the informatons. So $\vec{E}_g$ is the product of *N*, the density of the flow of informatons at *P*, with $\vec{s}_g$ their g-index:

$$\vec{E}_g = N.\vec{s}_g$$

According to the postulate of the emission of informatons, the magnitude of $\vec{s}_g$ is the elementary g-information quantity:

$$s_g = \frac{1}{K.\eta_0} = 6{,}18.10^{-60} m^3 s^{-1}$$

and the density of the flow of informatons at *P* is:

$$N = \frac{\dot{N}}{4\pi.r_0^2} \approx \frac{\dot{N}}{4.\pi.r^2} = \frac{K.m}{4\pi.r^2}$$

Taking into account that $\frac{1}{\eta_0.c^2} = \nu_0$, we obtain:

$$\vec{E}_g = -\frac{m}{4\pi.\eta_0.r^2}.\vec{e}_c - \{\frac{m}{4\pi.\eta_0.c.r^2}.v(t - \frac{r}{c}).sin\,\theta + \frac{\nu_0.m}{4\pi.r}.a(t - \frac{r}{c}).sin\,\theta\}.\vec{e}_{\perp c}$$

2. $\vec{B}_g$, the g-induction at *P*, is defined as the density of the cloud of β -information at that point. That density is the product of *n*, the density of the cloud of informations at *P* (number per unit volume) with $\vec{s}_\beta$, their β-index:

$$\vec{B}_g = n.\vec{s}_\beta$$

The β-index of an informaton refers to the information it carries about the state of motion of its emitter; it is defined as:

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c}$$

And the density of the cloud of informatons at *P* is related to *N*, the density of the flow of informatons at that point by: $n = \frac{N}{c}$.

So:

$$\vec{B}_g = n.\vec{s}_\beta = \frac{N}{c}.\frac{\vec{c} \times \vec{s}_g}{c} = \frac{\vec{c} \times (N.\vec{s}_g)}{c^2} = \frac{\vec{c} \times \vec{E}_g}{c^2}$$

And with the expression of that we have derived above under *1* we finally obtain:

$$\vec{B}_g = -\{\frac{\nu_0.m}{4.\pi.r^2}.v(t-\frac{r}{c}).sin\,\theta + \frac{\nu_0.m}{4.\pi.c.r}.a(t-\frac{r}{c}).sin\,\theta\}.\vec{e}_\varphi$$

From this it can be concluded that at a point *P*, sufficient far from the accelerated particle *m*, the components of its gravitational field are both transverse to the velocity of the informatons and they are proportional to $\frac{1}{r}$.

CHAPTER 6

THE MAXWELL-HEAVISIDE EQUATIONS

6 The Maxwell-Heaviside equations

The gravitational field is set up[1],[2],[3] by a given distribution of - whether or not moving - masses and it is defined by a vector field with two components: the "*g-field*" characterized by the vector $\vec{E}_g$ and the "*g-induction*" characterized by the vector $\vec{B}_g$. These components each have a value defined at every point of space and time and are thus, relative to an inertial reference frame ***O***, regarded as functions of the space and time coordinates.

Let us focus on the contribution to a gravitational field of one of its sources: a certain mass *m*. We focus, more specifically, on the contribution of *m* to the flow of g-information at an arbitrary point *P* in the field. That flow is made up of informatons that pass near *P* in a specific direction with velocity $\vec{c}$ and it is characterized by *N*, the rate per unit area at which these informatons cross an elementary surface perpendicular to the direction in which they move. The cloud of these informatons in the vicinity of *P* is characterized by the density *n*: *n* is the number of informatons per unit volume. *N* and *n* are linked by the relationship:

$$n = \frac{N}{c} \qquad (1)$$

The definition in chapter 2 of an informaton implies that every informaton that passes near *P* is characterized by two attributes that refer to its emitter: its g-index $\vec{s}_g$ and its β-index $\vec{s}_\beta$. s_g, the magnitude of the g-index is the elementary quantity of g-information. It is a fundamental physical constant. $\vec{s}_\beta$ refers to the state of motion of the source of the informaton and is defined by the relationship

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c} \qquad (2)$$

The informatons emitted by *m* that pass near *P* with velocity $\vec{c}$ contribute there to the *density of the g-information flow* with an amount $(N.\vec{s}_g)$. That vectoral quantity is the rate per unit area at which g-information at *P* crosses an elementary surface perpendicular to the direction in which it moves. It is the contribution of *m* to the g-field at *P*. We put

$$\vec{E}_g = N.\vec{s}_g$$

And the same informatons contribute there to the *density of the g-information cloud* with an amount $(n.\vec{s}_\beta)$. That vectoral quantity determines at P the amount of β-information per volume unit. It is the contribution of m to the g-induction at P. We put:

$$\vec{B}_g = n.\vec{s}_\beta$$

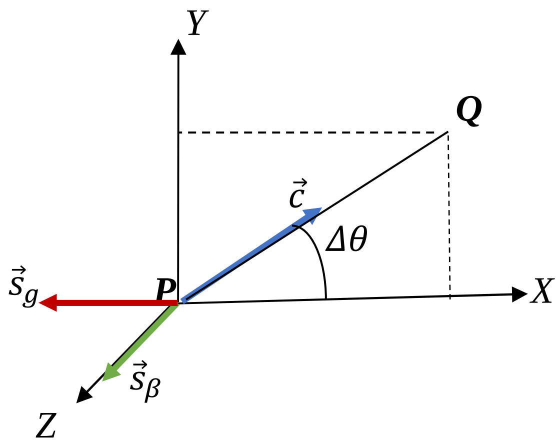

Fig 7

In fig 7, we consider the flow of informatons that - at the moment t - pass near P with velocity $\vec{c}$. They are completely defined by their attributes $\vec{s}_g$ and $\vec{s}_\beta$, respectively their g-index and their β-index. $\Delta\theta$ is their characteristic angle: the angle between the lines carrying $\vec{s}_g$ and $\vec{c}$ that is characteristic for the movement of the emitter.

The infinitesimal change of the attributes of an informaton at P between the moments t and $(t + dt)$, is governed by the kinematics of that information. An informaton that at the moment t passes at P is at the moment $(t + dt)$ at Q, with $PQ = c.dt$. This implies that the spatial variation of the attributes of an informaton between P and Q at the moment t equals the change in time of those attributes at P between the moment $(t - dt)$ and the moment t.

On the macroscopic level, this implies that there must be a relationship between the change in time of the gravitational field $(\vec{E}_g, \vec{B}_g)$ *at a point P and the spatial variation of that field in the vicinity of P.*

The intensity of the spatial variation of the components of the gravitational field at P is characterized by $div\vec{E}_g$, $div\vec{B}_g$, $rot\vec{E}_g$ and by $rot\vec{B}_g$ and the rate at which these components change in time by $\frac{\partial \vec{E}_g}{\partial t}$ and by $\frac{\partial \vec{B}_g}{\partial t}$.

From the above it can be concluded that it makes sense to investigate the relationships between the quantities that characterize the spatial variations of ($\vec{E}_g$,$\vec{B}_g$) and the rate's at which they change in time.

6.1 $\boldsymbol{div\vec{E}_g}$ - THE FIRST EQUATION IN FREE SPACE

In §3.1 and in §4.1 and it is shown that the physical fact that the rate at which g-information flows inward a closed empty space must be equal to the rate at which it flows outward, can be expressed as:

$$\oiint_S \vec{E}_g \, . \overrightarrow{dS} = 0$$

So (theorem of Ostrogradsky)[4]:

$$div\vec{E}_g = 0$$

In vacuum, the law of conservation of g-information can be expressed as followed:

(1) At a matter free point P of a gravitational field, the spatial variation of $\vec{E}_g$ *obeys the law:* $div\vec{E}_g = 0$

This is the first equation of Maxwell-Heaviside in vacuum.

Corollary: *At a matter free point P of a gravitational field* $\frac{\partial}{\partial t}[N.cos(\Delta\theta)] = 0$

Because[4]

$$div\vec{E}_g = div\left(N.\vec{s}_g\right) = grad(N).\vec{s}_g + N.div\left(\vec{s}_g\right) \qquad (3)$$

it follows from the first equation that:

$$grad(N).\vec{s}_g + N.div(\vec{s}_g) = 0$$

1. First we calculate: $grad(N).\vec{s}_g$.

Referring to fig 7:

$$grad(N) = \frac{N_Q - N_P}{PQ}.\vec{e}_c = \frac{N_Q - N_P}{c.dt}.\vec{e}_c$$

Because an informaton that at the moment *t* passes at *P* is at the moment (*t* + *dt*) at *Q*, (with *PQ* = *c.dt*).

$$\frac{N_Q - N_P}{dt} = \frac{N(t-dt) - N(t)}{dt} = -\frac{\partial N}{\partial t}$$

So:

$$grad(N) = -\frac{1}{c}.\frac{\partial N}{\partial t}.\vec{e}_c = -\frac{1}{c}.\frac{\partial N}{\partial t}.\frac{\vec{c}}{c}$$

And:

$$grad(N).\vec{s}_g = -\frac{1}{c^2}.\frac{\partial N}{\partial t}.\ \vec{c}.\vec{s}_g = \frac{1}{c}.\frac{\partial N}{\partial t}.s_g.cos(\Delta\theta) \qquad (4)$$

2. Next, we calculate: $N.div(\vec{s}_g)$

$$div(\vec{s}_g) = \frac{\oiint \vec{s}_g.\overrightarrow{dS}}{dV}$$

For that purpose, we calculate the double integral over the closed surface *S* formed by the infinitesimal surfaces *dS* that are at *P* and *Q* perpendicular to the flow of informatons (perpendicular to $\vec{c}$) and by the tube that connects the edges of these surfaces (and that is parallel to $\vec{c}$). *dV* =*c.dt.dS* is the infinitesimal volume enclosed by *S*:

$$div(\vec{s}_g) = \frac{\oiint \vec{s}_g.\overrightarrow{dS}}{dV} = \frac{s_g.dS.cos(\Delta\theta_P) - s_g.dS.cos(\Delta\theta_Q)}{dS.c.dt}$$

Because an informaton that at the moment t passes at P is at the moment (t + dt) at Q, (with $PQ = c.dt$):

$$\frac{cos(\Delta\theta_P) - cos(\Delta\theta_Q)}{dt} = \frac{\cos[\Delta\theta(t)] - \cos[\Delta\theta(t-dt)]}{dt} = \frac{\partial[\cos(\Delta\theta)]}{\partial t}$$

$$div(\vec{s}_g) = \frac{1}{c}.s_g.\frac{\partial\{cos(\Delta\theta)\}}{\partial t}$$

And:

$$N.div(\vec{s}_g) = \frac{N}{c}.s_g.\frac{\partial\{cos(\Delta\theta)\}}{\partial t} \qquad (5)$$

Substitution of (4) and (5) in (3) gives:

$$\frac{1}{c}.\frac{\partial N}{\partial t}.s_g.cos(\Delta\theta) + \frac{N}{c}.s_g.\frac{\partial\{cos(\Delta\theta)\}}{\partial t} = 0$$

Or:

$$\frac{\partial}{\partial t}[N.cos(\Delta\theta)] = 0 \qquad (\mathbf{6})$$

6.2 div$\vec{\mathbf{B}}_\mathbf{g}$ – THE SECOND EQUATION IN FREE SPACE

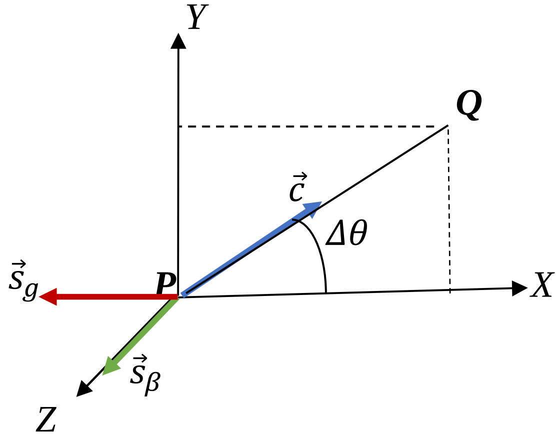

Fig 8

We refer to fig 8 and notice that:

$\vec{s}_g = -s_g.\vec{e}_x$ and $\vec{s}_\beta = \frac{\vec{c}\times\vec{s}_g}{c} = s_g.sin(\,\Delta\theta).\vec{e}_z$

From mathematics[4] we know:

$$div\vec{B}_g = div\left(n.\vec{s}_\beta\right) = grad(n).\vec{s}_\beta + n.div\left(\vec{s}_\beta\right) \qquad (7)$$

1. First we calculate: $grad(n).\vec{s}_\beta$

$grad(n).\vec{s}_\beta = 0$ because *grad(n)* is perpendicular to $\vec{s}_\beta$. Indeed *n* changes only in the direction of the flow of informatons, so *grad(n)* has the same orientation as $\vec{c}$:

2. Next we calculate: $n.div(\vec{s}_\beta)$

$$div(\vec{s}_\beta) = \frac{\oiint \vec{s}_\beta.\overrightarrow{dS}}{dV}$$

We calculate the double integral over the closed surface *S* formed by the infinitesimal surfaces *dS* = *dz.dy* that are at *P* and at *Q* perpendicular to the *X*-axis and by the tube that connects the edges of these surfaces.

Because $\vec{s}_\beta$ is oriented along the *Z*-axis the flux of $\vec{s}_\beta$ through the planes *dz.dy* and *dx.dz* is zero, while the fluxes through the planes *dx.dy* are equal and opposite. So we can conclude that:

$$div(\vec{s}_\beta) = \frac{\oiint \vec{s}_\beta.\overrightarrow{dS}}{dV} = 0$$

Both terms of the expression (7) of $div\vec{B}_g$ are zero, so $div\vec{B}_g = 0$, what implies (theorem of Ostrogradsky) that for every closed surface *S* in a gravitational field:

$$\oiint_S \vec{B}_g\,.\overrightarrow{dS} = 0$$

We conclude:

(2) *At a matter free point P of a gravitational field, the spatial variation of* $\vec{B}_g$ *obeys the law:* $div\vec{B}_g = 0$

This is the second equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that the β-index of an informaton is always perpendicular to both its g-index $\vec{s}_g$ and to its velocity $\vec{c}$.

6.3 $\boldsymbol{rot\vec{E}_g}$ - THE THIRD EQUATION IN FREE SPACE

The density of the flow of informatons that - at the moment *t* - passes near *P* with velocity $\vec{c}$ (fig 8) is defined as:

$$\vec{E}_g = N.\vec{s}_g = -N.s_g.\vec{e}_x$$

We know that[4]

$$rot\vec{E}_g = \left\{grad(N) \times \vec{s}_g\right\} + N.rot\left(\vec{s}_g\right) \qquad (8)$$

1. First we calculate: $\{grad(N) \times \vec{s}_g\}$

This expression describes the component of $rot\vec{E}_g$ caused by the spatial variation of *N* in the vicinity of *P* when $\Delta\theta$ remains constant.

N has the same value at all points of the infinitesimal surface that, at *P*, is perpendicular to the flow of informatons. So *grad(N)* is parallel to $\vec{c}$ and its magnitude is the increase of the magnitude of *N* per unit length. Thus, with *PQ = c.dt* , *grad(N)* is determined by:

$$grad(N) = \frac{N_Q - N_P}{PQ}.\frac{\vec{c}}{c} = \frac{N_Q - N_P}{c.dt}.\frac{\vec{c}}{c}$$

And:

$$grad(N) \times \vec{s}_g = \frac{N_Q - N_P}{c.dt}.\frac{\vec{c}}{c} \times \vec{s}_g = \frac{N_Q - N_P}{c.dt}.\vec{s}_\beta$$

The density of the flow of informatons at *Q* at the moment *t* is equal to the density of that flow at *P* at the moment (*t* - *dt*), so:

$$\frac{N_Q - N_P}{dt} = \frac{N(t - dt) - N(t)}{dt} = -\frac{\partial N}{\partial t}$$

And taking into account that :

$$\frac{N}{c} = n$$

we obtain:

$$grad(N) \times \vec{s}_g = -\frac{\partial n}{\partial t}.\vec{s}_\beta \qquad (9)$$

2. Next we calculate: $\{N.rot(\vec{s}_g)\ \}$

This expression describes the component of $rot\vec{E}_g$ caused by the spatial variation of $\Delta\theta$ - the orientation of the g-index - in the vicinity of P - when N remains constant. $(\Delta\theta)_P$ is the characteristic angle of the informatons that pass near P and $(\Delta\theta)_Q$ is the characteristic angle of the informatons that at the same moment pass near Q. (fig 9)

For the calculation of

$$rot(\vec{s}_g) = \frac{\oint \vec{s}_g.\overrightarrow{dl}}{dS}$$

with dS the encircled area, we calculate $\oint \vec{s}_g\,.\overrightarrow{dl}$ along the closed path $PQqpP$ that encircles dS: $dS = PQ.Pp = c.dt.Pp$. (PQ and qp are parallel to the flow of the informatons, Qq and pP are perpendicular to it).

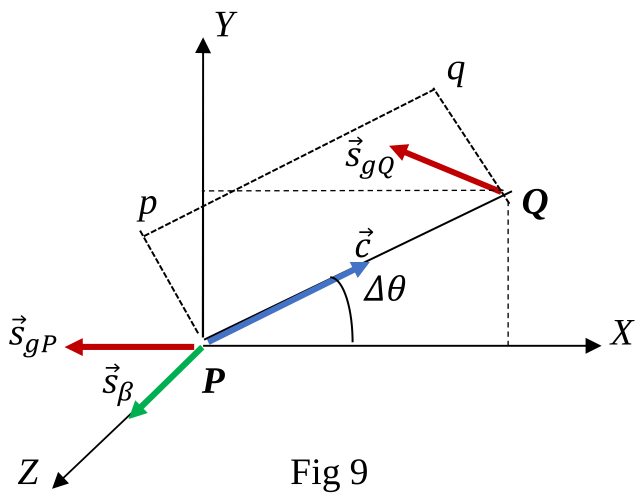

Fig 9

$$N.rot(\vec{s}_g) = N.\frac{s_g.sin[(\Delta\theta)_Q].Qq - s_g.sin[(\Delta\theta)_P)].pP}{c.dt.Pp}.\vec{e}_z$$

The characteristic angle of the informatons at Q at the moment t is equal to the characteristic angle of the informatons at P at the moment (t - dt), so:

$$N.rot(\vec{s}_g) = N.\frac{s_g.\sin[\Delta\theta(t - dt)].Qq - s_g.\sin[\Delta\theta(t)].pP}{c.dt.Pp}.\vec{e}_z$$

The rate at which sin($\Delta\theta$) in P changes at the moment t, is:

$$\frac{\partial\{sin(\Delta\theta)\}}{\partial t} = \frac{sin\{[\Delta\theta](t)\} - \sin\{[\Delta\theta](t - dt)\}}{dt}$$

And taking into account that

$$n = \frac{N}{c}$$

we obtain:

$$N.rot(\vec{s}_g) = -n.s_g.\frac{\partial\{sin(\Delta\theta)\}}{\partial t}.\vec{e}_z = -n.\frac{\partial}{\partial t}\{s_g.sin(\Delta\theta).\vec{e}_z\}$$

or

$$N.rot(\vec{s}_g) = -n.\frac{\partial\vec{s}_\beta}{\partial t} \qquad (10)$$

Combining the results (9) and (10), we obtain:

$$rot\vec{E}_g = grad(N) \times \vec{s}_g + N.rot(\vec{s}_g)$$

$$= -\left(\frac{\partial n}{\partial t}.\vec{s}_\beta + n.\frac{\partial\vec{s}_\beta}{\partial t}\right)$$

$$= -\frac{\partial(n.\vec{s}_\beta)}{\partial t} = -\frac{\partial\vec{B}_g}{\partial t} \qquad (\mathbf{11})$$

We conclude:

(3) At a matter free point P of a gravitational field, the spatial variation of $\vec{E}_g$ *and the rate at which* $\vec{B}_g$ *is changing are connected by the relation:*

$$rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

This is the third equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that any change of the product $n.\vec{s}_\beta$ at a point of a gravitational field is related to a spatial variation of the product $N.\vec{s}_g$ in the vicinity of that point.

The relation

$$rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

implies (theorem of Stokes[4]):

$$\oint \vec{E}_g \,.\overrightarrow{dl} = -\iint_S \frac{\partial \vec{B}_g}{\partial t}.\overrightarrow{dS} = -\frac{\partial}{\partial t}\iint_S \vec{B}_g \,.\overrightarrow{dS} = -\frac{\partial \Phi_B}{\partial t}$$

The orientation of the surface vector $\overrightarrow{dS}$ is linked to the orientation of the path on L by the "rule of the corkscrew". $\Phi_B = \iint_S \vec{B}_g \,.\overrightarrow{dS}$ is called the "β-information-flux through *S*".

So, *in a gravitational field, the rate at which the surface integral of* $\vec{B}_g$ *over a surface S changes is equal and opposite to the line integral of* $\vec{E}_g$ *over the boundary L of that surface.*

6.4 $\boldsymbol{rot\vec{B}_g}$ and $\boldsymbol{\frac{\partial \vec{E}_g}{\partial t}}$ - THE FOURTH EQUATION IN FREE SPACE

We consider again $\vec{E}_g$ and $\vec{B}_g$, the contributions of the informatons that - at the moment t – pass with velocity $\vec{c}$ near *P*, to the g-field and to the g-induction at that point. (fig 10).

$$\vec{E}_g = N.\vec{s}_g = -N.s_g.\vec{e}_x$$

and

$$\vec{B}_g = n.\vec{s}_\beta = n.\frac{\vec{c} \times \vec{s}_g}{c} = n.s_g.sin(\Delta\theta).\vec{e}_z$$

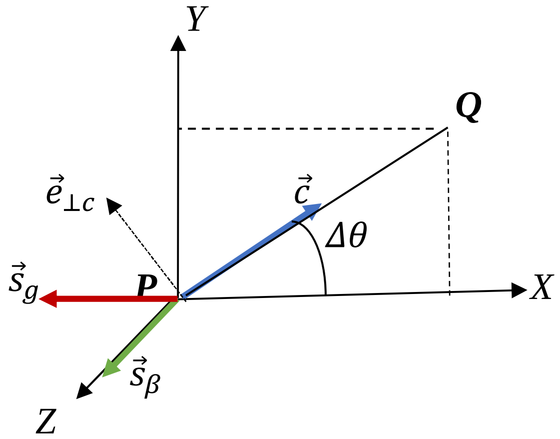

Fig 10

A. Let us calculate $rot\vec{B}_g$.

We know that[4]

$$rot\vec{B}_g = \{grad(n) \times \vec{s}_\beta\} + n.rot(\vec{s}_\beta) \qquad (12)$$

1. First we calculate: $\{grad(n) \times \vec{s}_\beta\}$

This expression describes the component of $rot\vec{B}_g$ caused by the spatial variation of *n* in the vicinity of *P* when $\Delta\theta$ remains constant.

n has the same value at all points of the infinitesimal surface that, at *P*, is perpendicular to the flow of informatons. So *grad(n)* is parallel to $\vec{c}$ and its magnitude is the increase of the magnitude of *n* per unit length.

With *PQ = c.dt* , *grad(n)* is determined by:

$$grad(n) = \frac{n_Q - n_P}{PQ}.\frac{\vec{c}}{c} = \frac{n_Q - n_P}{c.dt}.\frac{\vec{c}}{c}$$

The density of the cloud of informatons at *Q* at the moment *t* is equal to the density of that flow at *P* at the moment (*t* - *dt*), so:

$$\frac{n_Q - n_P}{dt} = \frac{n(t - dt) - n(t)}{dt} = -\frac{\partial n}{\partial t}$$

And

$$grad(n) = -\frac{1}{c}.\frac{\partial n}{\partial t}.\frac{\vec{c}}{c} = -\frac{1}{c}.\frac{\partial n}{\partial t}.\vec{e}_c$$

The vector $\{grad(n) \times \vec{s}_\beta\}$ is perpendicular to het plane determined by $\vec{c}$ and $\vec{s}_\beta$. So, it lies in the *XY*-plane and is there perpendicular to $\vec{c}$ forming an angle $\Delta\theta$ with the axis *OY*. Taking into account the definition of vectoral product we obtain:

$$grad(n) \times \vec{s}_\beta = -\frac{1}{c}.\frac{\partial n}{\partial t}.s_g.sin(\Delta\theta).(\vec{e}_c \times \vec{e}_z)$$

With

$$\vec{e}_c \times \vec{e}_z = -\vec{e}_{\perp c}$$

$$grad(n) \times \vec{s}_\beta = \frac{1}{c}.\frac{\partial n}{\partial t}.s_g.sin(\Delta\theta).\vec{e}_{\perp c}$$

And, taking into account that $n = \frac{N}{c}$, we obtain:

$$grad(n) \times \vec{s}_\beta = \frac{1}{c^2}.\frac{\partial N}{\partial t}.s_g.sin(\Delta\theta).\vec{e}_{\perp c} \qquad (13)$$

2. Next we calculate $\{n.rot(\vec{s}_\beta)\}$

This expression is the component of $rot\vec{B}_g$ caused by the spatial variation of $\vec{s}_\beta$ in the vicinity of *P* when *n* remains constant. For the calculation of

$$rot(\vec{s}_\beta) = \frac{\oint \vec{s}_\beta.\overrightarrow{dl}}{dS}.\vec{e}_{\perp c}$$

with *dS* the encircled area, we calculate $\oint \vec{s}_\beta.\overrightarrow{dl}$ along the closed path *PpqQP* that encircles *dS*: *dS*= *PQ.Pp* = *c.dt.Pp* (fig 11). (*PQ* and *qp* are parallel to the flow of the informatons, *Qq* and *pP* are perpendicular to it).

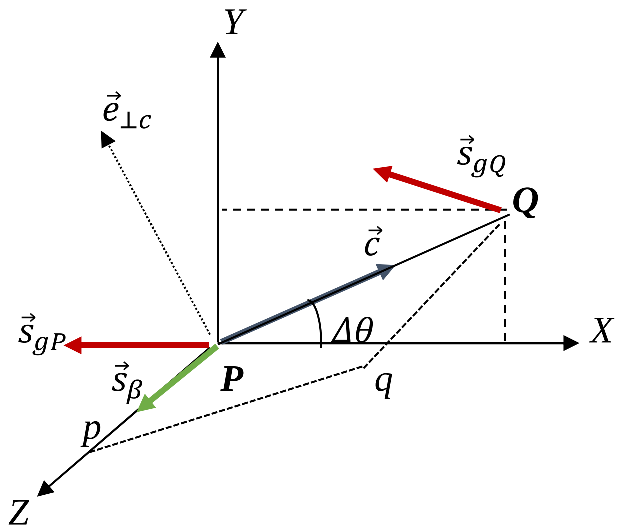

Fig 11

$$rot(\vec{s}_{\beta}) = \frac{\oint \vec{s}_{\beta}.\overrightarrow{dl}}{dS}.\vec{e}_{\perp c} = \frac{s_g.sin[(\Delta\theta)_P)].Pp - s_g.\sin\,[(\Delta\theta)_Q].qQ}{c.dt.Pp}\vec{e}_{\perp c}$$

The characteristic angle of the informatons at Q at the moment t is equal to the characteristic angle of the informatons at P at the moment ($t - dt$), so:

$$rot\left(\vec{s}_{\beta}\right) = \frac{\oint \vec{s}_{\beta}.\overrightarrow{dl}}{dS}.\vec{e}_{\perp c} = \frac{s_g.\{\sin[\Delta\theta(t)].Pp - s_g.\sin[\Delta\theta(t-dt)]\}.qQ}{c.dt.Pp}.\vec{e}_{\perp c}$$

The rate at which sin($\Delta\theta$) at P changes at the moment t, is:

$$\frac{\partial\{sin(\Delta\theta)\}}{\partial t} = \frac{sin\{(\Delta\theta)[t]\} - \sin\{(\Delta\theta)\,[t-dt]\}}{dt}$$

So:

$$rot\left(\vec{s}_{\beta}\right) = s_g.\frac{1}{c}.\frac{\partial[\sin(\Delta\theta)]}{\partial t}.\vec{e}_{\perp c}$$

And with

$$n = \frac{N}{c}$$

we finally obtain:

$$n.rot\left(\vec{s}_\beta\right) = s_g.\frac{1}{c^2}.N.\frac{\partial[\sin(\Delta\theta)]}{\partial t}.\vec{e}_{\perp c} \qquad (14)$$

Substituting the results (13) and (14) in (12) gives:

$$rot\vec{B}_g = \frac{1}{c^2}.s_g.\{\frac{\partial N}{\partial t}.sin(\Delta\theta) + N.\frac{\partial[\sin(\Delta\theta)]}{\partial t}\}.\vec{e}_{\perp c}$$

$$= \frac{1}{c^2}.s_g.\frac{\partial}{\partial t}[N.\sin(\Delta\theta)].\vec{e}_{\perp c} \qquad (\mathbf{15})$$

B. Now we calculate $\frac{\partial\vec{E}_g}{\partial t}$

We know that[4]:

$$\frac{\partial\vec{E}_g}{\partial t} = \frac{\partial N}{\partial t}.\vec{s}_g + N.\frac{\partial\vec{s}_g}{\partial t}$$

And that:

$$\vec{s}_g = -s_g.\vec{e}_x \qquad \text{and} \qquad \frac{\partial\vec{s}_g}{\partial t} = s_g.\frac{\partial(\Delta\theta)}{\partial t}.\vec{e}_y$$

So:

$$\frac{\partial\vec{E}_g}{\partial t} = -\frac{\partial N}{\partial t}.s_g.\vec{e}_x + N.s_g.\frac{\partial(\Delta\theta)}{\partial t}.\vec{e}_y$$

Taking into account:

$$\vec{e}_x = cos(\Delta\theta).\vec{e}_c - sin(\Delta\theta).\vec{e}_{\perp c} \quad \text{and} \quad \vec{e}_y = sin(\Delta\theta).\vec{e}_c + cos(\Delta\theta).\vec{e}_{\perp c}$$

we obtain:

$$\frac{\partial \vec{E}_g}{\partial t} = \left[-\frac{\partial N}{\partial t}.s_g.cos(\varDelta\theta) + N.s_g.\frac{\partial(\varDelta\theta)}{\partial t}.sin(\varDelta\theta)\right].\vec{e}_c$$

$$+\left[\frac{\partial N}{\partial t}.s_g.sin(\varDelta\theta) + N.s_g.\frac{\partial(\varDelta\theta)}{\partial t}.cos(\varDelta\theta)\right].\vec{e}_{\perp c}$$

or:

$$\frac{\partial \vec{E}_g}{\partial t} = s_g.\left\{-\frac{\partial}{\partial t}[N.cos(\varDelta\theta)\right].\vec{e}_c + \frac{\partial}{\partial t}[N.\sin(\varDelta\theta)].\vec{e}_{\perp c}\}$$

Taking into account (6), we find:

$$\frac{\partial \vec{E}_g}{\partial t} = s_g.\frac{\partial}{\partial t}[N.\sin(\varDelta\theta)].\vec{e}_{\perp c} \qquad (\mathbf{16})$$

C. From (15) an (16), we conclude:

$$rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t}$$

(4) *At a matter free point P of a gravitational field, the spatial variation of $\vec{B}_g$ and the rate at which $\vec{E}_g$ is changing are connected by the relation:*

$$rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t}$$

This is the fourth equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that any change of the product $N.\vec{s}_g$ at a point of a gravitational field is related to a spatial variation of the product $n.\vec{s}_g$ in the vicinity of that point.

This relation implies (theorem of Stokes): *In a gravitational field, the rate at which the surface integral of $\vec{E}_g$ over a surface S changes is proportional to the line integral of $\vec{B}_g$ over the boundary L of that surface:*

$$\oint \vec{B}_g.\overrightarrow{dl} = \frac{1}{c^2}\iint_S \frac{\partial \vec{E}_g}{\partial t}.\overrightarrow{dS} = \frac{1}{c^2}\frac{\partial}{\partial t}\iint_S \vec{E}_g.\overrightarrow{dS} = \frac{1}{c^2}\frac{\partial \Phi_G}{\partial t}$$

The orientation of the surface vector $\overrightarrow{dS}$ is linked to the orientation of the path on *L* by the "rule of the corkscrew". $\Phi_G = \iint_S \vec{E}_g\,.\overrightarrow{dS}$ is called the "g-information-flux through *S*".

6.5 THE MAXWELL-HEAVISIDE EQUATIONS

The volume-element at a point *P* inside a mass continuum is in any case an emitter of g-information and, if the mass is moving, also a source of β-information. According to §3.3, the instantaneous value of ρ_G - the mass density at *P* - contributes to the instantaneous value of $div\vec{E}_g$ at that point with an amount $-\frac{\rho_G}{\eta_0}$; and according to §4.6 the instantaneous value of $\vec{J}_G$ - the mass flow density - contributes to the instantaneous value of $rot\vec{B}_g$ at *P* with an amount $-\nu_0.\vec{J}_G$.

It is evident that at a point of a gravitational field - linked to an inertial reference frame ***O*** - one must take into account the contributions of the local values of $\rho_G(x,y,z;t)$ and of $\vec{J}_G(x,y,z;t)$. This results in the generalization and expansion of the laws at a mass free point. By superposition we obtain:

(1) *At a point P of a gravitational field, the spatial variation of* $\vec{E}_g$ *obeys the law:*

$$div\vec{E}_g = -\frac{\rho_G}{\eta_0}$$

In integral form:

$$\Phi_G = \oiint_S \vec{E}_g\,.\overrightarrow{dS} = -\frac{1}{\eta_0}.\iiint_G \rho_G.dV$$

(2) *At a point P of a gravitational field, the spatial variation of* $\vec{B}_g$ *obeys the law:*

$$div\vec{B}_g = 0$$

In integral form:

$$\Phi_B = \oiint_S \vec{B}_g\,.\overrightarrow{dS} = 0$$

(3) *At a point P of a gravitational field, the spatial variation of* $\vec{E}_g$ *and the rate at which* $\vec{B}_g$ *is changing are connected by the relation:*

$$rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

In integral form:

$$\oint \vec{E}_g \,.\, \overrightarrow{dl} = -\iint_S \frac{\partial \vec{B}_g}{\partial t} . \overrightarrow{dS} = -\frac{\partial}{\partial t}\iint_S \vec{B}_g \,.\, \overrightarrow{dS} = -\frac{\partial \Phi_B}{\partial t}$$

(4) *At a point P of a gravitational field, the spatial variation of* $\vec{B}_g$ *and the rate at which* $\vec{E}_g$ *is changing are connected by the relation:*

$$rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t} - \nu_0 . \vec{J}_G$$

In integral form:

$$\oint \vec{B}_g \,.\, \overrightarrow{dl} = \frac{1}{c^2}\iint_S \frac{\partial \vec{E}_g}{\partial t} . \overrightarrow{dS} - \nu_0 . \iint_S \vec{J}_g \,.\, \overrightarrow{dS} = \frac{1}{c^2} . \frac{\partial}{\partial t}\iint_S \vec{E}_g \,.\, \overrightarrow{dS} - \nu_0 . \iint_S \vec{J}_G \,.\, \overrightarrow{dS}$$

These are the laws of Heaviside-Maxwell or the laws of GEM.

6.6 CONCLUSION

The mathematical deductions of the laws of GEM confirm that these equations indicate that there is no causal link between $\boldsymbol{\vec{E}_g}$ and $\boldsymbol{\vec{B}_g}$. Therefore, *we must conclude that a gravitational field is a dual entity always having a "field-" and an "induction-" component simultaneously created by their common sources: time-variable masses and mass flows*[•]*.*

The GEM equations are analogue to Maxwell's equations in EM and it is proved[5] that these are consistent with special relativity. Thus, *the Maxwell-Heaviside equations are invariant under a Lorentz transformation* and GEM is consistent with special relativity. In this context it should be noted that the fact that the rate at which a material body emits informatons is independent of its velocity and completely defined by its rest mass m_0, implies that in equation (1) the value of

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[•] On the understanding that the induction-component equals zero if the source of the field is time independent.

$\rho_G = \frac{dm_0}{dV}$ depends on the state of motion – relative to the considered inertial reference system - of the mass element dm_0. Indeed in the case of a moving mass element, the Lorentz contraction must be taken into account in the determination of dV. Because a mass flow is made up of moving mass elements its density $\vec{J}_G$ also depends on the inertial reference frame in which it is considered. This implies that in equation (4) the expression of $\vec{J}_G$ also depends on the inertial reference frame.

In appendix B it is proven that the GEM equations are mathematically consistent.

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CHAPTER 7

THE GRAVITATIONAL INTERACTIONS

In the framework of the theory of informatons, the gravitational interactions are understood as the reaction of an object to the disturbance of its proper gravitational field by gravitational fields of other objects.

7.1 THE GRAVITATIONAL INTERACTION BETWEEN PARTICLES AT REST

We consider a set of point masses anchored in an inertial reference frame $\boldsymbol{O}$. They create and maintain a gravitational field that at each point of the space linked to $\boldsymbol{O}$ is completely determined by the vector $\vec{E}_g$**.** Each mass is "immersed" in a cloud of g-information. At every point, except at its own position, each mass contributes to the construction of that cloud.

Let us consider the mass *m* anchored at *P*. If the other masses were not there, then *m* would be at the centre of a perfectly spherical cloud of g-information. In reality this is not the case: the emission of g-information by the other masses is responsible for the disturbance of that "*characteristic symmetry*" of the proper g-field of *m*. Because $\vec{E}_g$ at *P* represents the intensity of the flow of g-information send to *P* by the other masses, the extent of disturbance of the characteristic symmetry in the immediate vicinity of *m* is determined by $\vec{E}_g$ at *P*.

If it was free to move, the point mass *m* could restore the characteristic symmetry of the g-information cloud in its immediate vicinity by accelerating with an amount $\vec{a} = \vec{E}_g$. Indeed, accelerating this way has the effect that the extern field disappears in the origin of the reference frame anchored to *m*. If it accelerates with an amount $\vec{a} = \vec{E}_g$, the mass would become "blind" for the g-information send to its immediate vicinity by the other masses, it "sees" only its proper spherical g-information cloud.

So, from the point of view of a particle at rest at a point *P* at a gravitational field $\vec{E}_g$, the characteristic symmetry of the g-information cloud in its immediate vicinity is conserved if it accelerates with an amount $\vec{a} = \vec{E}_g$. A point mass that is anchored in a gravitational field cannot accelerate. In that case it *tends* to move. These insight is expressed in the following postulate:

A particle anchored at a point of a gravitational field is subjected to a tendency to move in the direction defined by $\vec{E}_g$*, the g-field at that point. Once the anchorage is broken, the mass acquires a vectoral acceleration* $\vec{a}$ *that equals* $\vec{E}_g$*.*

7.2 THE GRAVITATIONAL FORCE – THE FORCE CONCEPT

A point mass *m*, anchored at a point *P* of a gravitational field, experiences an action because of that field, an action that is compensated by the anchorage.

1. That action is proportional to the extent to which the characteristic symmetry of the proper gravitational field of *m* in the immediate vicinity of *P* is disturbed by the extern g-field, thus to the value of $\vec{E}_g$ at *P*.

2. It depends also on the magnitude of *m*. Indeed, the g-information cloud created and maintained by *m* is more compact if *m* is greater. That implies that the disturbing effect on the spherical symmetry around *m* by the extern g-field $\vec{E}_g$ is smaller when *m* is greater. Thus, to impose the acceleration $\vec{a} = \vec{E}_g$, the action of the gravitational field on *m* must be greater if *m* is greater.

We can conclude that the action that tends to accelerate a point mass *m* in a gravitational field must be proportional to $\vec{E}_g$, the g-field to which the mass is exposed; and to *m*, the magnitude of the mass. We represent that action by $\vec{F}_G$ and we call this vectoral quantity "the force developed by the g-field on the mass" or the *gravitational force* on *m*. We define it by the relation:

$$\vec{F}_G = m.\vec{E}_g$$

A mass anchored at a point *P* cannot accelerate, what implies that the effect of the anchorage must compensate the gravitational force. It cannot be otherwise than that the anchorage exerts an action on *m* that is exactly equal and opposite to the gravitational force. That action is called a *reaction force*.

Between the gravitational force on a mass *m* and the local field strength exists the following relationship:

$$\vec{E}_g = \frac{\vec{F}_G}{m}$$

So, the acceleration imposed to the mass by the gravitational force is:

$$\vec{a} = \frac{\vec{F}_G}{m}$$

Considering that the gravitational force is nothing but a special force, we can conclude that this relation can be generalized.

The relation between a force $\vec{F}$ and the acceleration $\vec{a}$ that it imposes to a free mass m is:

$$\vec{F} = m.\vec{a}$$

7.3 NEWTONS LAW OF UNIVERSAL GRAVITATION

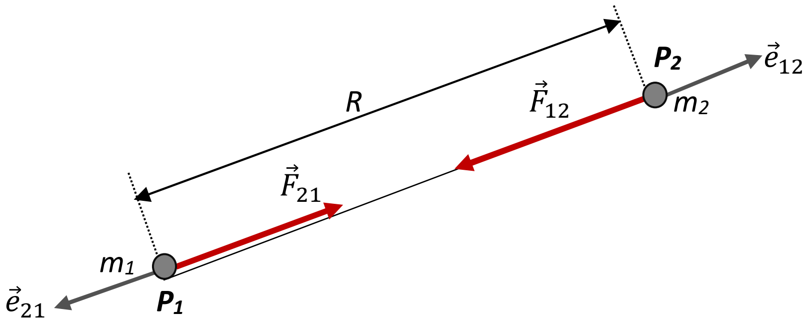

Fig 12

In fig 12 we consider two particles with (rest) masses m_1 and m_2 anchored at the points P_1 and P_2 of an inertial reference frame.

1. m_1 creates and maintains a gravitational field that at P_2 is defined by the g-field:

$$\vec{E}_{g2} = -\frac{m_1}{4.\pi.\eta_0.R^2}.\vec{e}_{12}$$

We show that this result is embedded in the GEM description of gravity.

The first GEM-equation - that is the mathematical expression of the conservation of g-information - states that the flux of the gravitational field through an arbitrary closed surface S is determined by the enclosed mass m_{in} according to the law:

$$\oiint \vec{E}_g\,.\overrightarrow{dS} = -\frac{m_{in}}{\eta_0} \qquad (1)$$

Let us apply this equation to an hypothetical sphere S with radius R centered on P_1.

1. Because of the symmetry, $\vec{E}_g$ is at every point of that sphere perpendicular to its surface and has the same magnitude. So, at an arbitrary point *P* of the sphere, $\vec{E}_g$ can be expressed as

$$\vec{E}_g = E_{gr}.\vec{e}_r$$

where $\vec{e}_r$ and E_{gr} respectively are the unit vector and the component (with constant magnitude) of $\vec{E}_g$ in the direction of $\overrightarrow{P_1P}$.

Further, at any point of the surface of the sphere: $\overrightarrow{dS} = dS.\bar{e}_r$.

With this information we calculate $\oiint_S \vec{E}\,.\overrightarrow{dS}$:

$$\oiint_S \vec{E}_g\,.\overrightarrow{dS} = \oiint_S E_{gr}.\vec{e}_r\,.dS.\vec{e}_r = \oiint_S E_{gr}.dS = E_{gr}.\oiint_S dS = E_{gr}.4\pi R^2 \quad (2)$$

2. The enclosed mass is m_1, so

$$m_{in} = m_1 \qquad (3)$$

Taking into account (2) and (3), (1) becomes:

$$E_{gr}.4\pi R^2 = -\frac{m_1}{\eta_0}$$

We conclude: at a point *P* at a distance *R* from P_1 the gravitational field is pointing to P_1 and determined by:

$$\vec{E}_g = E_{gr}.\vec{e}_r = -\frac{m_1}{4\pi\eta_0 R^2}.\vec{e}_r$$

In particular at the point P_2:

$$\vec{E}_{g2} = -\frac{m_1}{4.\pi.\eta_0.R^2}.\vec{e}_{12}$$

If m_2 was free, according to the postulate of the gravitational interaction it would accelerate with an amount $\vec{a}$:

$$\vec{a} = \vec{E}_{g2}$$

So the gravitational field of m_1 exerts a "gravitational force" on m_2:

$$\vec{F}_{12} = m_2.\vec{a} = m_2.\vec{E}_{g2} = -\frac{m_1.m_2}{4.\pi.\eta_0.R^2}.\vec{e}_{12}$$

In a similar manner we find $\vec{F}_{21}$:

$$\vec{F}_{21} = -\frac{m_1.m_2}{4.\pi.\eta_0.R^2}.\vec{e}_{21} = -\vec{F}_{12}$$

This is the mathematical expression of "Newton's law of universal gravitation"[1]:

The force between any two particles having masses m_1 and m_2 separated by a distance R is an attraction acting along the line joining the particles and has the magnitude

$$F = G.\frac{m_1.m_2}{R^2} = \frac{1}{4\pi\eta_0}.\frac{m_1.m_2}{R^2}$$

$G = \frac{1}{4\pi\eta_0}$ is a universal constant having the same value for all pairs of particles.

7.4 THE GRAVITATIONAL INTERACTION BETWEEN MOVING OBJECTS

We consider a number of point masses moving relative to an inertial reference frame ***O***. They create and maintain a gravitational field that at each point of the space linked to ***O*** is defined by the vectors $\vec{E}_g$ and $\vec{B}_g$. Each mass is "immersed" in a cloud of informatons carrying both g- and β-information. At each point, except at its own position, each mass contributes to the construction of that cloud.

Let us consider the mass *m* that, at the moment *t*, goes through the point *P* with velocity $\vec{v}$.

1. If the other masses were not there $\vec{E}'_g$ - the g-field in the immediate vicinity of *m* (the proper g-field of *m*) - would, according to §4.2, be symmetric relative to the carrier line of the vector $\vec{v}$. This results from the fact that the g-indices of the informatons emitted by *m* during the time interval (*t* - *Δt*, *t* + *Δt*) are all directed to that line. In reality that symmetry is disturbed by the g-information that the other masses send to P. $\vec{E}_g$, the instantaneous value of the g-field at P, defines the extent to which this occurs.

2. If the other masses were not there $\vec{B}'_g$ - the g-induction near *m* (the proper g-induction of *m*) - would , according to §4.4, "rotate" around the carrier line of the vector $\vec{v}$. The vectors defining the pseudo-gravitational-field $E''_g = \vec{v} \times \vec{B}'_g$ defined by the vector product of $\vec{v}$ with the g-induction that characterizes the proper β-field of *m*, would - just like $\vec{E}'_g$ - be symmetric relative to the carrier line of the vector $\vec{v}$. In reality this symmetry is disturbed by the β-information send to *P* by the other masses. The vector product $(\vec{v} \times \vec{B}_g)$ of the instantaneous values of the velocity of *m* and of the g-induction at *P*, characterizes the extent to which this occurs.

So, the *characteristic symmetry* of the cloud of g-information around a moving mass (the proper gravitational field) is in the immediate vicinity of *m* disturbed by $\vec{E}_g$ regarding the proper g-field; and by $(\vec{v} \times \vec{B}_g)$ regarding the proper β-induction.

If it was free to move, the point mass *m* could restore the characteristic symmetry in its immediate vicinity by accelerating with an amount $\vec{a}' = \vec{E}_g + (\vec{v} \times \vec{B}_g)$ relative to its proper inertial reference frame• $\boldsymbol{O'}$. In that manner it would become "blind" for the disturbance of symmetry of the gravitational field in its immediate vicinity. These insights form the basis of the following postulate.

A particle m, moving with velocity $\vec{v}$ in a gravitational field $(\vec{E}_g, \vec{B}_g)$, tends to become blind for the influence of that field on the symmetry of its proper gravitational field. If it is free to move, it will accelerate relative to its proper inertial reference frame with an amount $\vec{a}'$:

$$\vec{a}' = \vec{E}_g + (\vec{v} \times \vec{B}_g)$$

• The proper inertial reference frame ***O'*** of the particle *m* is the reference frame that at each moment *t* moves relative to ***O*** with the same velocity as *m*.

7.5 THE GRAVITATIONAL FORCE LAW

The action of the gravitational field ($\vec{E}_g, \vec{B}_g$) on a point mass that is moving with velocity $\vec{v}$ relative to the inertial reference frame ***O***, is called the *gravitational force* $\vec{F}_G$ on that mass. In extension of §7.2 we define $\vec{F}_G$ as:

$$\vec{F}_G = m_0. \left[\vec{E}_g + (\vec{v} \times \vec{B}_g)\right]$$

m_0 is the rest mass of the point mass: it is the mass that determines the rate at which it emits informatons in the space linked to ***O***. If it is free to move, the effect of $\bar{F}_G$ on the point mass *m* is that it will be accelerated relative to the proper inertial reference frame ***O'*** with an amount $\vec{a}'$:

$$\vec{a}' = \vec{E}_g + (\vec{v} \times \vec{B}_g)$$

This acceleration can be decomposed in a tangential ($\vec{a}'_T$) and a normal component ($\vec{a}'_N$):

$$\vec{a}'_T = a'_T.\vec{e}_T \qquad \text{and} \qquad \vec{a}'_N = a'_N.\vec{e}_N$$

where $\vec{e}_T$ and $\vec{e}_N$ are the unit vectors, respectively along the tangent and along the normal to the path of the point mass in ***O'*** (and in ***O***).

We express a'_T and a'_N in function of the characteristics of the motion in the inertial reference system ***O*** [2]:

$$a'_T = \frac{1}{(1-\beta^2)^{\frac{3}{2}}}.\frac{dv}{dt} \qquad \text{and} \qquad a'_N = \frac{v^2}{R.\sqrt{1-\beta^2}}$$

(*R* is the radius of curvature of the path in ***O***, and that radius in ***O'*** is $R\sqrt{1-\beta^2}$.)

The gravitational force is:

$$\vec{F}_G = m_0.\vec{a}' = m_0.(a'_T.\vec{e}_T + a'_N.\vec{e}_N)$$

$$= m_0.\left[\frac{1}{(1-\beta^2)^{\frac{3}{2}}}.\frac{dv}{dt}.\vec{e}_T + \frac{1}{(1-\beta^2)^{\frac{1}{2}}}.\frac{v^2}{R}.\vec{e}_N\right] = \frac{d}{dt}\left[\frac{m_0}{\sqrt{1-\beta^2}}.\vec{v}\right]$$

Finally with:

$$\frac{m_0}{\sqrt{1-\beta^2}}.\vec{v} = \vec{p}$$

We obtain:

$$\vec{F}_G = \frac{d\vec{p}}{dt}$$

$\vec{p}$ is the linear momentum of the particle relative to the inertial reference frame ***O***. It is a measure for its inertia, for its ability to persist in its dynamic state .

7.6 THE INTERACTION BETWEEN TWO MOVING PARTICLES

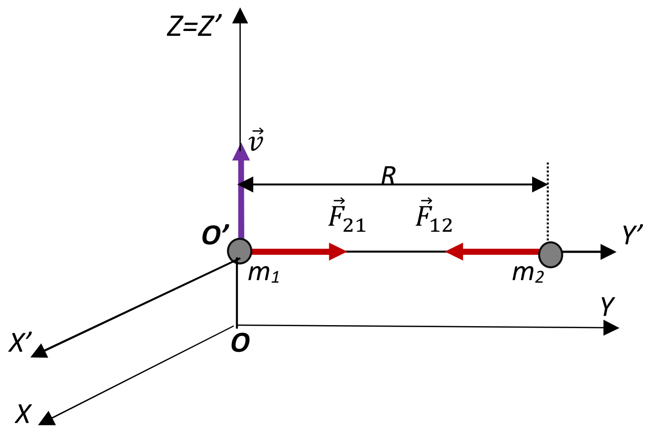

Fig 13

Two particles with rest masses m_1 and m_2 (fig 13) are anchored in the inertial reference frame ***O'*** that is moving relative to the inertial reference frame ***O*** with constant velocity $\vec{v} = v.\bar{e}_z$. The distance between the masses is R.

Relative to ***O'*** the particles are at rest. According to Newton's law of universal gravitation, they exert on each other equal but opposite forces:

$$F' = F'_{12} = F'_{21} = G.\frac{m_{.1}.m_2}{R^2} = \frac{1}{4.\pi.\eta_0}.\frac{m_1.m_2}{R^2}$$

Relative to ***O*** both masses are moving with constant speed v in the direction of the Z-axis. From the transformation equations between an inertial frame ***O*** and another inertial frame ***O'***, in which a point mass experiencing a force F' is instantaneously at rest, we can immediately deduce the force F that the point masses exert on each other in ***O*** [2] :

$$F = F_{12} = F_{21} = F'.\sqrt{1 - (\frac{v}{c})^2} = F'.\sqrt{1 - \beta^2}$$

We will now show that also this form of Newton's law of universal gravitation perfectly can be deduced in the framework of GEM.

1. According to §4.4, at a point *P* - whose position is determined by the time dependent position vector $\vec{r}$ - the gravitational field $(\vec{E}_g, \vec{B}_g)$ of a particle with rest mass m_0 that is moving with constant velocity $\vec{v} = v.\bar{e}_z$ along the *Z*-axis of the inertial reference frame ***O*** (fig 14) is determined by:

$$\vec{E}_g = -\frac{m_0}{4\pi\eta_0 r^3}.\frac{1 - \beta^2}{(1 - \beta^2.sin^2\,\theta)^{\frac{3}{2}}}.\vec{r} = -\frac{m_0}{4\pi\eta_0 r^2}.\frac{1 - \beta^2}{(1 - \beta^2.sin^2\,\theta)^{\frac{3}{2}}}.\vec{e}_r$$

$$\vec{B}_g = -\frac{m_0}{4\pi\eta_0 c^2.r^3}.\frac{1 - \beta^2}{(1 - \beta^2.sin^2\,\theta)^{\frac{3}{2}}}.(\vec{v} \times \vec{r})$$

With $\beta = \frac{v}{c}$, the dimensionless speed of m_0.

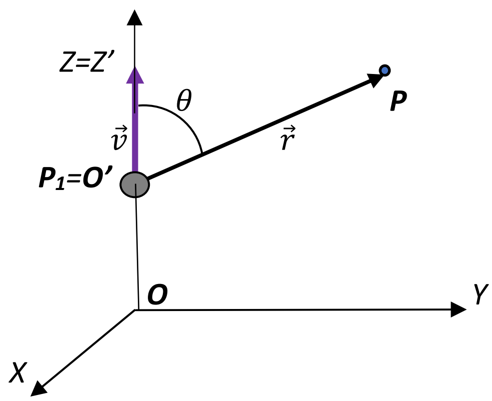

Fig 14

2. In the inertial reference frame ***O*** of fig 13, the masses m_1 and m_2 are moving in the direction of the *Z*-axis with speed v. m_2 moves through the gravitational field generated by m_1, and m_1 moves through that generated by m_2.

According the above formulas, the magnitude of the gravitational field created and maintained by m_1 at the position of m_2 is determined by:

$$E_{g2} = \frac{m_1}{4\pi\eta_0 R^2} . \frac{1}{\sqrt{1-\beta^2}}$$

$$B_{g2} = \frac{m_1}{4\pi\eta_0 R^2} . \frac{1}{\sqrt{1-\beta^2}} . \frac{v}{c^2}$$

And according to the force law $\vec{F}_G = m_0 . \left[\vec{E}_g + (\vec{v} \times \vec{B}_g)\right]$, F_{12}, the magnitude of the force exerted by the gravitational field $(\vec{E}_{g2}, \vec{B}_{g2})$ on m_2 - this is the attraction force of m_1 on m_2 - is:

$$F_{12} = m_2 . (E_{g2} - v . B_{g2})$$

After substitution:

$$F_{12} = \frac{1}{4\pi\eta_0} . \frac{m_1 m_2}{R^2} . \sqrt{1-\beta^2} = F'_{21} . \sqrt{1-\beta^2}$$

In the same way we find:

$$F_{21} = \frac{1}{4\pi\eta_0} . \frac{m_1 m_2}{R^2} . \sqrt{1-\beta^2} = F'_{12} . \sqrt{1-\beta^2}$$

We conclude that the moving masses attract each other with a force:

$$F = F_{12} = F_{21} = F' . \sqrt{1-\beta^2}$$

This result perfectly agrees with that based on S.R.T.

We also can conclude that the component of the gravitational force due to the g-induction is β^2 times smaller than that due to the g-field. This implies that, for speeds much smaller than the speed of light, the effects of the β-information are masked.

It can be shown that the β-information emitted by moving gravitating objects is responsible for deviations (as the advance of Mercury Perihelion) of the real orbits of planets with respect to these predicted by the classical theory of gravitation[3].

7.7 THE EQUIVALENCE MASS-ENERGY

The instantaneous value of the force $\vec{F}$ that acts on a particle with rest mass m_0, that freely moves relative to the inertial reference frame ***O*** with velocity $\vec{v}$, and the linear momentum $\vec{p} = m.\vec{v}$ of that particle are related by:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

The elementary vectoral displacement $d\vec{r}$ of m_0 during the elementary time interval dt is:

$$d\vec{r} = \vec{v}.dt$$

And the elementary work done by $\vec{F}$during dt is[1]:

$$dW = \vec{F}.d\vec{r} = \vec{F}.\vec{v}.dt = \vec{v}.d\vec{p}$$

With

$$\vec{p} = m.\vec{v} = \frac{m_0}{\sqrt{1 - (\frac{v}{c})^2}}.\vec{v}$$

this becomes:

$$dW = \frac{m_0.v.dv}{\left[1 - (\frac{v}{c})^2\right]^{\frac{3}{2}}} = d\left[\frac{m_0}{\sqrt{1 - (\frac{v}{c})^2}}.c^2\right] = d(m.c^2)$$

The work done on the moving particle equals, by definition, the increase of the energy of the mass. So, $d(m.c^2)$ is the increase of the energy of the mass and $m.c^2$ is the energy represented by the mass. We can conclude:

A particle with relativistic mass m is equivalent to an amount of energy of $m.c^2$.

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CHAPTER 8

GRAVITATIONAL WAVES

We will show that the existence of gravitational waves is embedded in GEM. In the framework of the theory of informatons a gravitational wave is understood as the macroscopic manifestation of the fact that the "train" of informatons emitted by an oscillating source and travelling with the speed of light in a certain direction is a spatial sequence of informatons whose characteristic angle is harmonically fluctuating along the "train" what implies that the component of their g-index perpendicular to their velocity $\vec{c}$ and their β-index fluctuate harmonically in space. Gravitational waves transport gravitational energy because some of the informatons that constitute the "train" are carriers of energy. They are called gravitons.

8.1 THE WAVE EQUATION

In free space - where $\rho_G = \vec{J}_G = 0$ - the GEM equations are:

1. $div\vec{E}_g = 0$

2. $div\vec{B}_g = 0$

3. $rot\vec{E}_g = -\dfrac{\partial\vec{B}_g}{\partial t}$

4. $rot\vec{B}_g = \dfrac{1}{c^2}\dfrac{\partial\vec{E}_g}{\partial t}$

To attempt a solution of a group of simultaneous equations, it is usually a good plan to separate the various functions of space to arrive at equations that give the distributions of each.

It follows from (3):

$$rot\left(rot\vec{E}_g\right) = -rot\left(\frac{\partial\vec{B}_g}{\partial t}\right) \qquad (3')$$

Because[1] $rot(rot\vec{F}) = grad(div\vec{F}) - \nabla^2\vec{F}$, where $\nabla^2$is the Laplacian, (3') leads to:

$$grad(div\vec{E}_g) - \nabla^2\vec{E}_g = -rot(\frac{\partial\vec{B}_g}{\partial t}) = -\frac{\partial}{\partial t}(rot\vec{B}_g)$$

And taking into account (1) and (4):

$$\nabla^2\vec{E}_g = \frac{1}{c^2}.\frac{\partial^2\vec{E}_g}{\partial t^2} \qquad (5)$$

This is the general form of the wave equation. This form applies as well to the g-induction, as is readily shown by taking first the rotor of (4) and then substituting (2) and (3):

$$\nabla^2\vec{B}_g = \frac{1}{c^2}.\frac{\partial^2\vec{B}_g}{\partial t^2} \qquad (5')$$

Solutions of this equation describe how disturbances of the gravitational field propagate as waves with speed c.

To illustrate this we consider the special case of space variation in one dimension only. If we take the *x*-component of (5) and have space variations only in the *z*-direction, the equation becomes simply:

$$\frac{\partial^2 E_{gx}}{\partial z^2} = \frac{1}{c^2}.\frac{\partial^2 E_{gx}}{\partial t^2}$$

This equation has a general solution of the form

$$E_{gx} = f_1\left(t - \frac{z}{c}\right) + f_2\left(t + \frac{z}{c}\right) \qquad (6)$$

The first term of (6) represents the wave or function f_1 traveling with velocity c and unchanged form in the positive *z*-direction, the second term represents the wave or function f_2 traveling with velocity c and unchanging form in the negative *z*-direction.

8.2 GRAVITATIONAL WAVE EMITTED BY A HARMONICALLY OSCILLATING PARTICLE

In fig 15 we consider a point mass *m* that harmonically oscillates around the origin of the inertial reference frame ***O*** with frequency $\nu = \frac{\omega}{2.\pi}$. At the moment *t* it passes at P_1. We suppose that the speed of the particle is always much smaller than the speed of light and that it is described by:

$$v(t) = V.\cos \omega\, t$$

The elongation *z(t)* and the acceleration *a(t)* are then expressed as:

$$z(t) = \frac{V}{\omega}.\cos(\,\omega t - \frac{\pi}{2}) \qquad \text{and} \qquad a(t) = \omega.V.\cos(\,\omega t + \frac{\pi}{2})$$

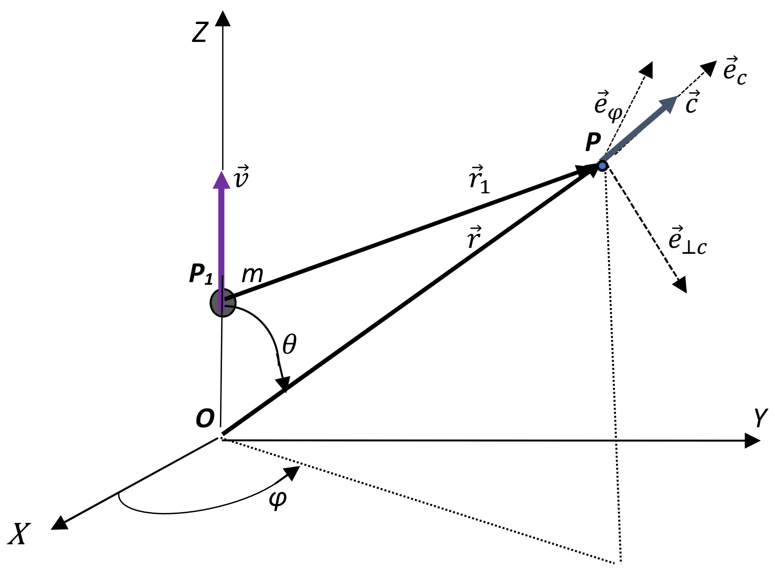

Fig 15

We restrict our considerations about the gravitational field of *m* to points *P* that are sufficiently far away from the origin *O*. Under that condition we can posit that the fluctuation of the length of the vector $\overrightarrow{P_1P} = \vec{r}_1$ is very small relative to the length of the time-independent position vector $\vec{r}$, that defines the position of *P* relative to the origin *O*. In other words: we assume that the amplitude of the oscillation is very small relative to the distances between the origin and the points *P* on which we focus.

8.2.1 The transversal gravitational field of a harmonically oscillating particle

Starting from the complex quantity $\bar{V} = V.e^{j.0}$ - that is representing $v(t)$ - $\bar{E}_{g\perp c}$, the complex representation of the time dependent part of the transversal component of $\vec{E}_g$, and $\bar{B}_{g\varphi}$, the complex representation of $B_{g\varphi}$, at P follows immediately from §5.2:

$$\bar{E}_{g\perp c} = -\frac{m.\bar{V}}{4\pi}.e^{-j.k.r}.(\frac{1}{\eta_0.c.r^2} + \frac{j.\omega.\nu_0}{r}).sin\,\theta$$

$$\bar{B}_{g\varphi} = -\frac{\nu_0.m.\bar{V}}{4\pi}.e^{-j.k.r}.(\frac{1}{r^2} + \frac{j.k}{r}).sin\,\theta$$

where $k = \frac{\omega}{c}$ the phase constant. Note that $\bar{B}_{g\varphi} = \frac{\bar{E}_{g\perp c}}{c}$.

Thus, relative to ***O***, $B_{g\varphi}$ and the time dependent part of $E_{g\perp c}$ are expressed as functions of the space and time coordinates as:

$$\begin{aligned} B_{g\varphi}(r,\theta;t) &= \frac{E_{g\perp c}(r,\theta;t)}{c} \\ &= \frac{\nu_0.m.V.sin\,\theta\,.\sqrt{1+k^2r^2}}{4\pi r^2}.cos(\,\omega t - kr + \Phi + \pi) \end{aligned}$$

with $tg\Phi = kr$.

So, an harmonically oscillating particle emits a transversal "gravitomagnetic" wave that propagates out of the mass with the speed of light:

In points at a great distance from the oscillating mass, specifically there where $r >> \frac{1}{k} = \frac{c}{\omega}$, this expression asymptotically equals:

$$B_{g\varphi} = \frac{E_{g\perp c}}{c} = \frac{\nu_0.k.m.V.sin\,\theta}{4\pi r}.sin(\,\omega t - kr)$$

$$= \frac{\nu_0.m.\omega.V.sin\,\theta}{4\pi cr}.sin(\,\omega t - kr)$$

$$= -\frac{v_0 . m . a\left(t - \frac{r}{c}\right) . \sin\theta}{4\pi c r}$$

The intensity of the “far gravitational field” is inversely proportional to *r*, and is determined by the component of the acceleration of *m*, that is perpendicular to the direction of $\vec{e}_c$.

8.2.2 The longitudinal gravitational field of a harmonically oscillating particle

The oscillation of the point mass m along the Z-axis is responsible for the existence of a fluctuation of $r_0 = P_0P$, the distance travelled by the informatons at the moment t when they pass near P. Within the framework of our approximations:

$$r_0(t) \approx r_1(t) \approx r - z(t - \frac{r}{c}) . \cos\theta = r . \left\{1 - \frac{z(t - \frac{r}{c})}{r} . \cos\theta\right\}$$

and

$$(\frac{1}{r_0})^2 \approx \frac{1}{r^2} . (1 + 2 . \frac{z(t - \frac{r}{c})}{r} . \cos\theta)$$

From §5.2, it follows:

$$E_{gc} = -\frac{m}{4 . \pi . \eta_0 . r^2} - \frac{m}{4 . \pi . \eta_{.0} . r^3} . 2 . z(t - \frac{r}{c}) . \cos\theta$$

So $\bar{E}_{gc}$, the complex representation of the time dependant part of the longitudinal gravitationel field is:

$$\bar{E}_{gc} = -\frac{m . \bar{V}}{4\pi} . e^{-jkr} . \frac{2}{j . \omega . \eta_0 . r^3} . \cos\theta$$

We conclude that an harmonically oscillating point mass emits a longitudinal gravitational wave that - relative to the position of the mass - expands with the speed of light:

$$E_{gc}(r,\theta;t) = \frac{m.V}{4.\pi.\eta_0.c.k}.\frac{2}{r^3}.sin(\omega t - kr)$$

Because its amplitude is proportional to $\frac{1}{r^3}$, at a great distance from the emitter the longitudinal field can be neglected relative to the transversal.

8.3 GRAVITATIONAL WAVE EMITTED BY AN OBJECT WITH VARIABLE REST MASS

Another phenomenon that is the source of a gravitational wave is the conversion of rest mass into energy (what per example happens in the case of radioactive processes). To illustrate this, let us - relative to an inertial reference frame - consider a particle with rest mass m_0 that - due to intern instability - during the period $(0, \Delta t)$ emits EM radiation.

This implies that that particle during that time interval is emitting electromagnetic energy U_{EM} carried by photons (and gravitational energy U_{GEM}[•] carried by gravitons) that propagate with the speed of light. Between the moment $t = 0$ and the moment $t = \Delta t$, the rest mass of the particle is, because of this event, decreasing *with* an amount $\frac{U_{EM}(+U_{GEM})}{c^2}$ from the value m_0 to the value m_0'. *B*ecause the gravitational field is determined by the rest mass, this implies that if $t < 0$ the source of the gravitational field of the particle is m_0 and for $t > \Delta t$ it is m_0'. It follows that at the moment t the gravitational field at a point P at a distance $r>c.t$ is proportional to m_0, and at a point at a distance $r < c.(t - \Delta t)$ to m_0'.

During the period $(t, t+\Delta t)$ the gravitational field at a point at a distance $r = c.t$ changes from the situation where it is determined by m_0 to the situation where it is determined by m_0'. So, the conversion of rest mass of an object into radiation is the cause of a kink in the gravitational field of that object, a kink that with the speed of light - together with the emitted radiation - propagates out of the object.

We can conclude that the conversion of (a part of) the rest mass of an object into radiation goes along with the emission by that object of a gravitational wave.

[•] negligible in first approximation

The effect of the decrease - during the time interval $(0, \Delta t)$ - of the rest mass of a point mass on the magnitude of its g-field E_g at the point P at a distance r is shown in the plot of fig. 16.

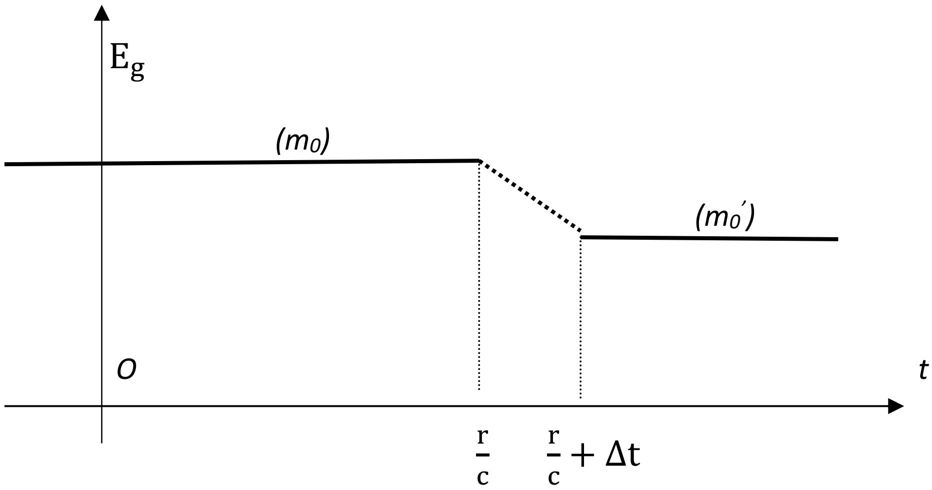

Fig 16

1. Until the moment $t = \frac{r}{c}$, the effect of the conversion of rest mass into radiation has not yet reached *P*. So, during the period $(0, \frac{r}{c})$ the quantity of mass-energy enclosed by an hypothetical sphere with radius r centered on the particle is still m_0 (the remaining part of the rest mass + all the radiation that during the mentioned period has arisen from the conversion of rest mass). From the first GEM equation it follows:

$$E_g = \frac{m_0}{4\pi\eta_0 . r^2}$$

2. From the moment $t = \frac{r}{c} + \Delta t$, the radiation generated by the conversion of rest mass has left the space enclosed by the hypothetical sphere with radius r, that from that moment only contains the remaining rest mass m_0'. From the first GEM equation it follows:

$$E_g = \frac{{m_0}'}{4\pi\eta_0 . r^2}$$

3. During the time interval $(\frac{r}{c}, \frac{r}{c} + \Delta t)$, the mass-energy enclosed by the hypothetical sphere with radius *r* is decreasing (not necessary linearly) because mass-energy flows out in the form of radiation. So, during that period E_g at P is decreasing.

8.4 ON THE DETECTION OF GRAVITATIONAL WAVES USING AN INTERFEROMETER

Let x and y be the directions of the arms L_1 and L_2 of an interferometer, and let *z* be the direction perpendicular to the plane defined by the arms. We consider the (optimized) situation where a uniform plane gravitational wave $(\vec{E}_g, \vec{B}_g)$ of sinusoidal form is travelling in the z-direction. We assume that the gravitational field $\vec{E}_g$ is in the *x*-direction and that the gravitational induction $\vec{B}_g$ is in the *y*-direction. If E_{MAX} is the amplitude of the gravitational field, than - according to GEM - $\vec{E}_g$ is given in magnitude by: $E_g = E_{MAX}.sin(\omega t - kz)$ with $k = \frac{\omega}{c}$, and the magnitude of $\vec{B}_g$ is given by: $B_g = \frac{E_g}{c}$.

When that gravitational wave is falling on the plane of the interferometer, the gravitational field $\vec{E}_g$ - being in the direction of L_1 - will induce a longitudinal mechanical wave in the tube of the arm L_1 what will result in a (very slight) oscillation of the mirror at the end. The mirror at the end of the arm L_2 will not react on $\vec{E}_g$ because that field is perpendicular to L_2. So, the effective length of the light beam that is travelling through L_1 will differ (in the manner of an oscillation) from the effective length of the light beam that is travelling through L_2, and the detector will record that the outgoing and reflected beams are out of phase. It is clear that this can be generalized and that *we can conclude that, according to GEM ,the interferometer will reacts on a gravitational wave.*

8.5 THE ENERGY RADIATED BY A HARMONICALLY OSCILLATING PARTICLE

8.5.1 Poynting's theorem

In free space a gravitational field is completely defined by the vectoral functions $\vec{E}_g(x, y, z; t)$ and $\vec{B}_g(x, y, z; t)$. It can be shown[2] that the spatial area G enclosed by the surface *S* - at the moment *t* - contains an amount of energy given by the expression:

$$U = \iiint_G (\frac{\eta_0 . E_g^2}{2} + \frac{B_g^2}{2\nu_0}).dV$$

The rate at which the energy escapes from *G* is:

$$-\frac{\partial U}{\partial t} = -\iiint_V (\eta_0 . \vec{E}_g . \frac{\partial \vec{E}_g}{\partial t} + \frac{1}{\nu_0} . \vec{B}_g . \frac{\partial \vec{B}_g}{\partial t}) . dV$$

According to the third law of GEM:

$$rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

and according to the fourth law:

$$rot\frac{\vec{B}_g}{\nu_0} = \eta_0 . \frac{\partial \vec{E}_g}{\partial t}$$

So:

$$-\frac{\partial U}{\partial t} = \iiint_G (\frac{\vec{B}_g}{\nu_0} . rot\vec{E}_g - \vec{E}_g . rot\frac{\vec{B}_g}{\nu_0}) . dV = \iiint_G div(\frac{\vec{E}_g \times \vec{B}_g}{\nu_0}) . dV$$

By application of the theorem of Ostrogradsky[1]: $\iiint_G div\vec{F} . dV = \oiint_S \vec{F} . \overrightarrow{dS}$, we can rewrite this as:

$$-\frac{\partial U}{\partial t} = \oiint_S \frac{\vec{E}_g \times \vec{B}_g}{\nu_0} . \overrightarrow{dS}$$

from which we can conclude that the expression

$$\frac{\vec{E}_g \times \vec{B}_g}{\nu_0} . \overrightarrow{dS}$$

defines the rate at which energy flows in the sense of the positive normal through the surface element *dS* at *P* .

So, the density of the energy flow at P is:

$$\frac{\vec{E}_g \times \vec{B}_g}{\nu_0}$$

This vectoral quantity is called the “Poynting’s vector”. It is represented by $\vec{P}$:

$$\vec{P} = \frac{\vec{E}_g \times \vec{B}_g}{\nu_0}$$

The amount of energy transported through the surface element *dS* in the sense of the positive normal during the time interval *dt* is:

$$dU = \frac{\vec{E}_g \times \vec{B}_g}{\nu_0} . \overrightarrow{dS} . dt$$

8.5.2 The energy radiated by a harmonically oscillating particle – gravitons

In §8.2 it is shown that an harmonically oscillating point mass *m* radiates a gravitomagnetic wave that at a far point *P* is defined by (see fig 13):

$$\vec{E}_g = E_{g\perp c} . \vec{e}_{\perp c} = \frac{\nu_0 . m . \omega . V . \sin\theta}{4\pi r} . \sin(\omega t - kr) . \vec{e}_{\perp c}$$

$$\vec{B}_g = B_{g\varphi} . \vec{e}_\varphi = \frac{\nu_0 . m . \omega . V . \sin\theta}{4\pi c r} . \sin(\omega t - kr) . \vec{e}_\varphi$$

The instantaneous value of Poynting's vector at *P* is:

$$\vec{P} = \frac{\nu_0 . m^2 . \omega^2 . V^2 . \sin^2\theta}{16 . \pi^2 . c . r^2} . \sin^2(\omega t - kr) . \vec{e}_c$$

The amount of energy that, during one period T , flows through the surface element *dS* that at *P* is perpendicular to the direction of the movement of the informatons, is:

$$dU = \int_0^T P . dt . dS = \frac{\nu_0 . m^2 . \omega^2 . V^2 . \sin^2\theta}{16 . \pi^2 . c . r^2} . \frac{T}{2} . dS$$

And, with $\omega = \frac{2.\pi}{T} = 2 . \pi . \nu$:

$$dU = \frac{\nu_0 . m^2 . V^2 . \sin^2\theta}{8c} . \nu . \frac{dS}{r^2}$$

$\frac{dS}{r^2} = d\Omega$ is the solid angle under which dS is "seen" from the origin. So, the oscillating mass radiates per unit of solid angle in the direction θ, per period, an amount of energy u_Ω:

$$u_\Omega = \frac{\nu_0 . m^2 . V^2 . \sin^2\theta}{8c} . \nu \qquad (1)$$

This quantity is greatest in the direction perpendicular to the movement of the mass ($\theta = 90°$) and it is proportional to the frequency of the wave, thus proportional to the frequency at which the mass is oscillating.

We posit that the energy radiated by an oscillating point mass travels through space in the form of particle-like packets of energy, called "gravitons" and that the energy U_g transported by a graviton is proportional to the frequency of the oscillator, so:

$$Ug = h' . \nu \qquad (2)$$

h' plays the role of Planck's constant in electromagnetism.

A graviton can be understood as an information transporting a quantum of energy.

From (1) and (2), it follows that the number of gravitons emitted per period and per unit of solid angle in the direction θ by an oscillating point mass *m* is:

$$N_{g\Omega} = \frac{u_\Omega}{h' . \nu} = \frac{\nu_0 . m^2 . V^2 . \sin^2\theta}{8 . h' . c}$$

what is independent of the duration of a period.

If we assume that the number of gravitons and the number of photons emitted by an oscillating charged particle (e. g. an electron) are of the same order of magnitude, it turns out that the value of h' depends on the nature of the emitter and that the energy of a graviton is many orders smaller than that of a photon [2].

8.6 CONCLUSION

The existence of gravitational waves is embedded in the GEM description of gravity. According to the theory of informatons a gravitational wave is the macroscopic manifestation of the fact that the "train" of informatons emitted by an oscillating source and travelling with the speed of light in a certain direction is a spatial sequence of informatons whose characteristic angle is harmonically fluctuating along the “train” what implies that the component of their g-index perpendicular to their velocity $\vec{c}$ and their β-index fluctuate harmonically in space. Gravitational waves transport gravitational energy because some of the informatons that constitute the “train” are carriers of energy. They are called gravitons. However, the energy quantum carried by a graviton is small in such a way that it is very difficult to give experimental evidence of its existence.

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APPENDIX 1

THE GRAVITATIONAL FIELD OF AN OBJECT MOVING WITH CONSTANT VELOCITY AND THE GEM EQUATIONS

In fig. A-1 we consider the gravitational field of a particle with rest mass m_0 that is moving with constant velocity $\vec{v} = v.\vec{e}_z$ along the Z-axis of an inertial reference frame ***O***. At the moment when the particle passes at the origin O, we set $t = 0$.

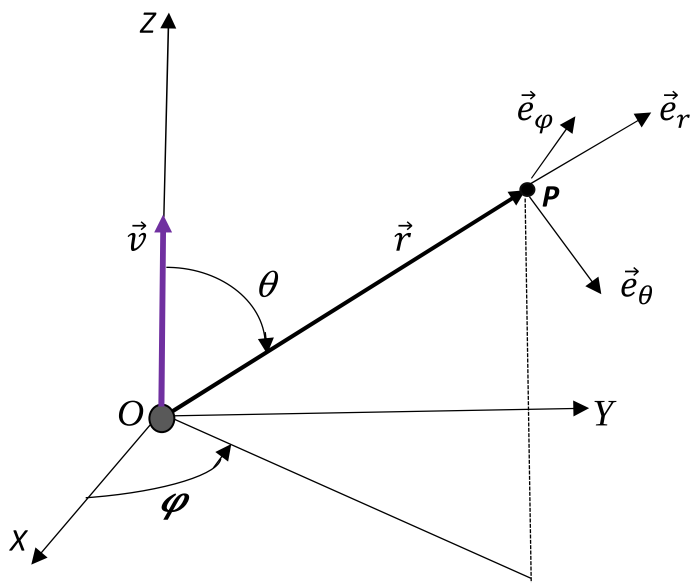

Fig. A-1

According to §4.5 the gravitational field of that particle at P is completely defined, in spherical coordinates (r, θ, φ), by:

$$\vec{E}_g = N.\vec{s}_g = -\frac{m_0}{4\pi\eta_0 r^2}.\frac{1-\beta^2}{(1-\beta^2.sin^2\,\theta)^{\frac{3}{2}}}.\vec{e}_r$$

$$\vec{B}_g = n.\vec{s}_\beta = -\frac{\nu_0.m_0}{4\pi r^2}.\frac{1-\beta^2}{(1-\beta^2.sin^2\,\theta)^{\frac{3}{2}}}.v.\sin\theta\,.\vec{e}_\varphi$$

We will verify that $(\vec{E}_g, \vec{B}_g)$ satisfy the Maxwell-Heaviside equations at an arbitrary point P:

1. $div\vec{E}_g = 0$ 2. $div\vec{B}_g = 0$

3. $rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$ 4. $rot\vec{B}_g = \frac{1}{c^2}.\frac{\partial \vec{E}_g}{\partial t}$

1. $div\vec{E}_g = 0$

From mathematics we know that:

$$div\vec{E}_g = \frac{1}{r^2}.\frac{\partial}{\partial r}(r^2.E_{gr}) + \frac{1}{r.\sin\theta}.\frac{\partial}{\partial\theta}(\sin\theta\,.E_{g\theta}) + \frac{1}{r.\sin\theta}.\frac{\partial E_{g\varphi}}{\partial\varphi}$$

With:

$$E_{gr} = -\frac{m_0}{4\pi\eta_0 r^2}.\frac{1-\beta^2}{(1-\beta^2.\sin^2\theta)^{\frac{3}{2}}} \quad \text{and} \quad E_{g\theta} = E_{g\varphi} = 0$$

We find: $div\vec{E}_g = 0$

2. $div\vec{B}_g = 0$

One can prove this in the same way as *1*.

3. $rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$

From mathematics we know that:

$$rot\vec{E}_g = \frac{1}{r.\sin\theta}.\left[\frac{\partial}{\partial\theta}\left(E_{g\varphi}.\sin\theta\right) - \frac{\partial E_{g\theta}}{\partial\varphi}\right].\vec{e}_r$$

$$+\frac{1}{r}.\left[\frac{1}{\sin\theta}.\frac{\partial E_{gr}}{\partial\varphi}-\frac{\partial}{\partial r}\left(r.E_{g\varphi}\right)\right].\vec{e}_{\theta}$$

$$+\frac{1}{r}.\left[\frac{\partial}{\partial r}(r.E_{g\theta})-\frac{\partial E_{gr}}{\partial\theta}\right].\vec{e}_{\varphi}$$

With:

$$E_{gr}=-\frac{m_0}{4\pi\eta_0 r^2}.\frac{1-\beta^2}{(1-\beta^2.\sin^2\theta)^{\frac{3}{2}}};\ E_{g\theta}=E_{g\varphi}=0\ \text{ and }\ \beta^2=\frac{v^2}{c^2}\eta_0.\nu_0.v^2$$

We find:

$$rot\vec{E}_g=3\frac{\nu_0 m_0}{4\pi r^3}.\frac{1-\beta^2}{(1-\beta^2.\sin^2\theta)^{\frac{5}{2}}}.v^2.\sin\theta.\cos\theta.\vec{e}_{\varphi}\qquad(1)$$

Next we calculate $\frac{\partial\vec{B}_g}{\partial t}$.

Taking into account that from the kinematics of the particle along the *Z*-axis, it follows that:

$$\frac{\partial r}{\partial t}=-v.\cos\theta\qquad and\qquad \frac{\partial\theta}{\partial t}=\frac{v.\sin\theta}{r}$$

We find:

$$\frac{\partial\vec{B}_g}{\partial t}=-3\frac{\nu_0 m_0}{4\pi r^3}.\frac{1-\beta^2}{(1-\beta^2.\sin^2\theta)^{\frac{5}{2}}}.v^2.\sin\theta.\cos\theta.\vec{e}_{\varphi}\qquad(2)$$

From (1) and (2) it follows:

$$rot\vec{E}_g=-\frac{\partial\vec{B}_g}{\partial t}$$

4. $\boldsymbol{rot\vec{B}_g=\frac{1}{c^2}.\frac{\partial\vec{E}_g}{\partial t}}$

One can prove this in the same way as *3*.

APPENDIX 2

THE GEM EQUATIONS ARE MATHEMATICALLY CONSISTENT

At a point P of a gravitational field - where ρ_G is the mass density and $\vec{J}_G$ is the density of the mass flow - $\vec{E}_g$ and $\vec{B}_g$ must obey to the GEM equations (the Maxwell-Heaviside equations):

1. $div\vec{E}_g = -\dfrac{\rho_G}{\eta_0}$

2. $div\vec{B}_g = 0$

3. $rot\vec{E}_g = -\dfrac{\partial \vec{B}_g}{\partial t}$

4. $rot\vec{B}_g = \dfrac{1}{c^2}\dfrac{\partial \vec{E}_g}{\partial t} - \nu_0.\vec{J}_G$

And: $\eta_0.\nu_0 = \frac{1}{c^2}$

We will prove that these equations are mathematically consistent.

1 THE CASE OF AN OBJECT WITH INVARIABLE REST MASS

Because $div(rot\vec{F}) = 0$, it follows from (4) that:

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left(div\vec{E}_g\right) - \nu_0.div\vec{J}_G = 0 \qquad (4')$$

Substituting (1) in (4') gives:

$$-\frac{1}{c^2\eta_0}.\frac{\partial \rho_G}{\partial t} - \nu_0.div\vec{J}_G = 0$$

And with $\frac{1}{c^2\eta_0} = \nu_0$, we obtain from (4'):

$$\frac{\partial \rho_G}{\partial t} + div\vec{J}_G = 0 \qquad (4")$$

(4”) is nothing else but the expression of the law of mass conservation. Indeed:

- The rate at which mass is flowing out form a closed surface S is:

$$\oiint_S \vec{J}_G.\overrightarrow{dS} \qquad (A)$$

- The rate of the decrease of the mass enclosed by S is (V is the volume enclosed by S):

$$-\frac{\partial}{\partial t}\iiint_V \rho_G.dV = \iiint_V \left(-\frac{\partial \rho_G}{\partial t}\right).dV \qquad (B)$$

Because of the law of mass conservation (A) = (B), so

$$\oiint_S \vec{J}_G.\overrightarrow{dS} = \iiint_V (-\frac{\partial \rho_G}{\partial t}).dV \qquad (5)$$

Ostrogradsky’s theorem (divergence theorem) states that

$$\oiint_S \vec{F}.\overrightarrow{dS} = \iiint_V div\vec{F}.dV$$

Substituting in (5) gives:

$$\iiint_V div\vec{J}_G.dV = \iiint_V (-\frac{\partial \rho_G}{\partial t}).dV$$

It follows:

$$div\vec{J}_G = -\frac{\partial \rho_G}{\partial t}$$

Or:

$$\frac{\partial \rho_G}{\partial t} + div\vec{J}_G = 0$$

We conclude that - in a system with invariable rest mass - the GEM equations of the gravitational field are in line with the law of mass conservation.

2. THE CASE OF AN OBJECT WITH VARIABLE REST MASS

Let us consider - relative to an inertial reference frame - an object with rest mass m_0 that - due to intern instability - during the period ($0, \Delta t)$ emits EM radiation. This implies that that object during that time interval is emitting electromagnetic energy U_{EM} carried by photons (+ gravitomagnetic energy• U_{GEM} carried by gravitons) that propagate with the speed of light. Because of that event, from the moment $t = \Delta t$ the rest mass of the particle is decreased with an amount $\frac{U_{EM}+(U_{GEM})}{c^2}$ to the value m_0'.

Consider the surface S enclosing the object in whole or in part (V is the volume enclosed by S). At a moment $0 < t < \Delta t$:

- The rate of the decrease of the enclosed mass is:

$$-\frac{\partial}{\partial t}\iiint_V \rho_G dV = \iiint_V \left(-\frac{\partial \rho_G}{\partial t}\right).dV \qquad \text{(A)}$$

- $\vec{J}_G$, the density of the mass flow out from the enclosed volume at a point P of S has two components:

  1. $\vec{J}_{G1}$ describing the outflow of massive mass;
  2. $\vec{J}_{G2}$ describing the outflow of mass in the form of energy. If we represent the density of that energy flow by $\vec{S}$: $\vec{J}_{G2} = \frac{\vec{S}}{c^2}$

  So:

$$\vec{J}_G = \vec{J}_{G1} + \vec{J}_{G2} = \vec{J}_{G1} + \frac{\vec{S}}{c^2}$$

,

and the rate at which mass-energy is flowing out from the closed surface S is:

$$\oiint_S \vec{J}_G.\overrightarrow{dS} \qquad \text{(B)}$$

(A) = (B) because of the law of mass-energy conservation, so

$$\oiint_S \vec{J}_G.\overrightarrow{dS} = \iiint_V (-\frac{\partial \rho_G}{\partial t}).dV$$

and

• negligible in first approximation

$$div\vec{J}_G = -\frac{\partial \rho_G}{\partial t} \qquad or \qquad \frac{\partial \rho_G}{\partial t} + div\vec{J}_G = 0$$

We conclude that in the case of a system with variable rest mass, the GEM equations of the gravitational field are in line with the law of mass-energy conservation.

APPENDIX 3

THE THEORY OF INFORMATONS AND ELECTROMAGNETISM

The theory of informatons unifies gravitation with electromagnetism. Indeed, with the theory of informatons it is also possible to explain the phenomena and the laws of electromagnetism[1],[2],[3]. It is sufficient to add the following rule to the postulate of the emission of informatons:

C. *Informatons emitted by an electrically charged particle (a "point charge" q) at rest in an inertial reference frame, carry an attribute referring to the charge of the emitter, namely the e-index. e-indices are represented as $\vec{s}_e$ and defined by:*

1. *The e-indices are radial relative to the position of the emitter. They are centrifugal when the emitter carries a positive charge (q = +Q) and centripetal when the charge of the emitter is negative (q = -Q).*

2. *s_e, the magnitude of an e-index depends on Q/m, the charge per unit of mass of the emitter. It is defined by:*

$$s_e = \frac{1}{K.\varepsilon_0}.\frac{Q}{m} = 8{,}32.10^{-40}.\frac{Q}{m}N.m^2.s.C^{-1}$$

where $\varepsilon_0 = 8{,}85.10^{-12}F/m$ is the permittivity constant.

Consequently, the informatons emitted by a moving point charge *q* have at the fixed point *P* - defined by the time dependant position vector $\vec{r}$ (see fig 5) - two attributes that are in relation with the fact that *q* is a *moving* point *charge*, namely their e-index $\vec{s}_e$ and their b-index $\vec{s}_b$:

$$\vec{s}_e = \frac{q}{m}.\frac{1}{K.\varepsilon_0}.\vec{e}_r = \frac{q}{m}.\frac{1}{K.\varepsilon_0}.\frac{\vec{r}}{r} \quad \text{and} \quad \vec{s}_b = \frac{\vec{c}\times\vec{s}_e}{c} = \frac{\vec{v}\times\vec{s}_e}{c}$$

Macroscopically, these attributes manifest themselves at *P* as, respectively the *electric field strength* (the *e-field*) $\vec{E}$ and the *magnetic induction* (the *b-induction*) $\vec{B}$.

References

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